Solar Physics –

Dynamics of the Sun's Interior

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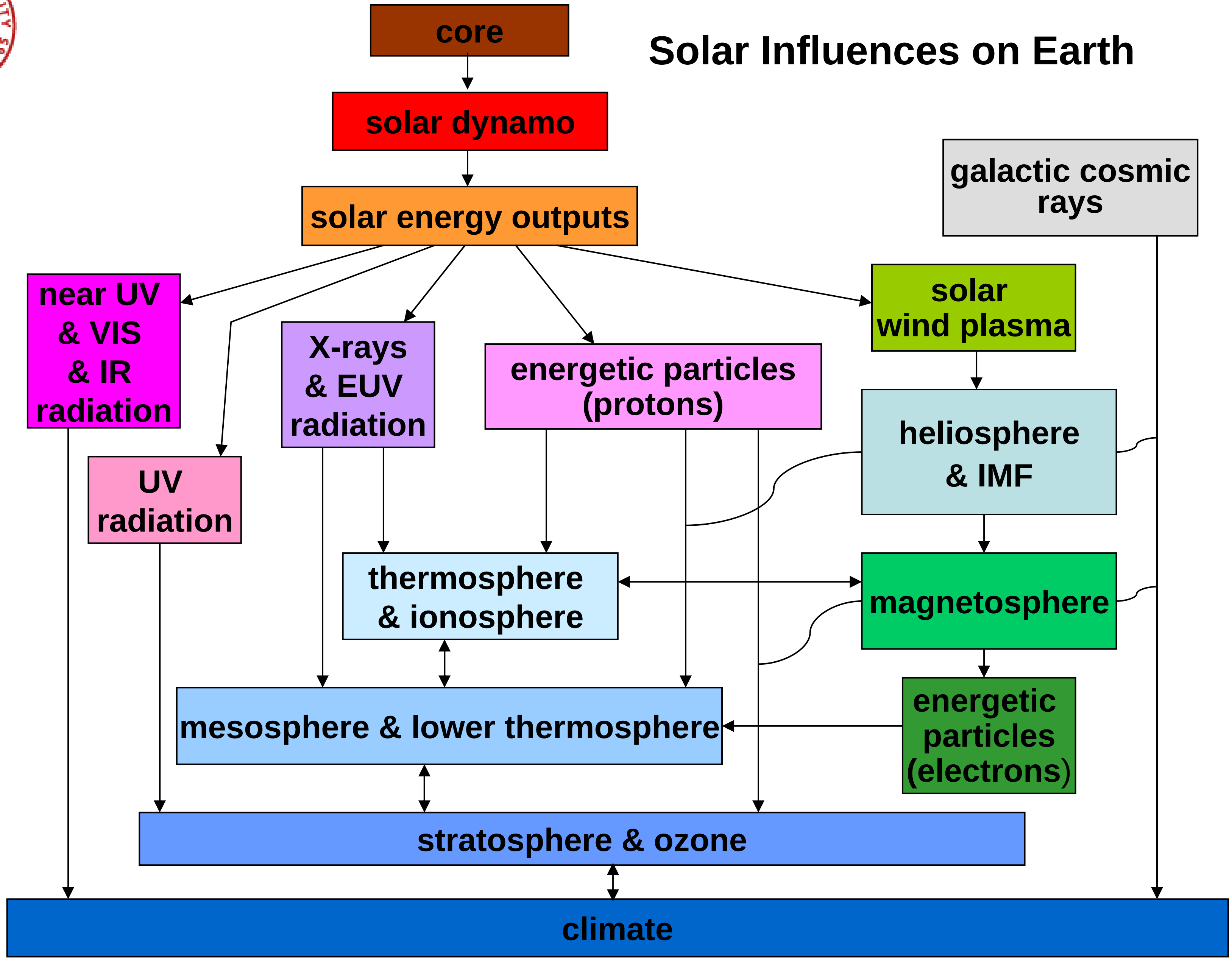
Solar Influences on Earth



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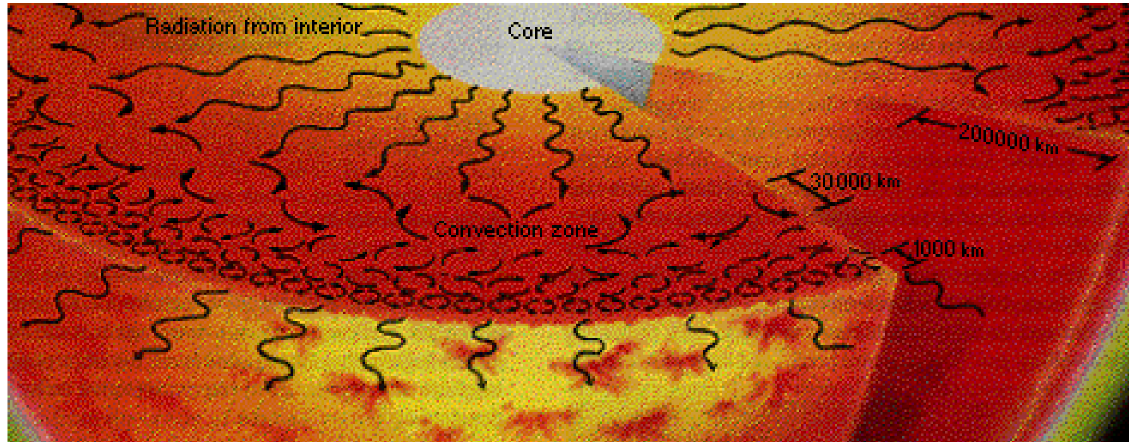
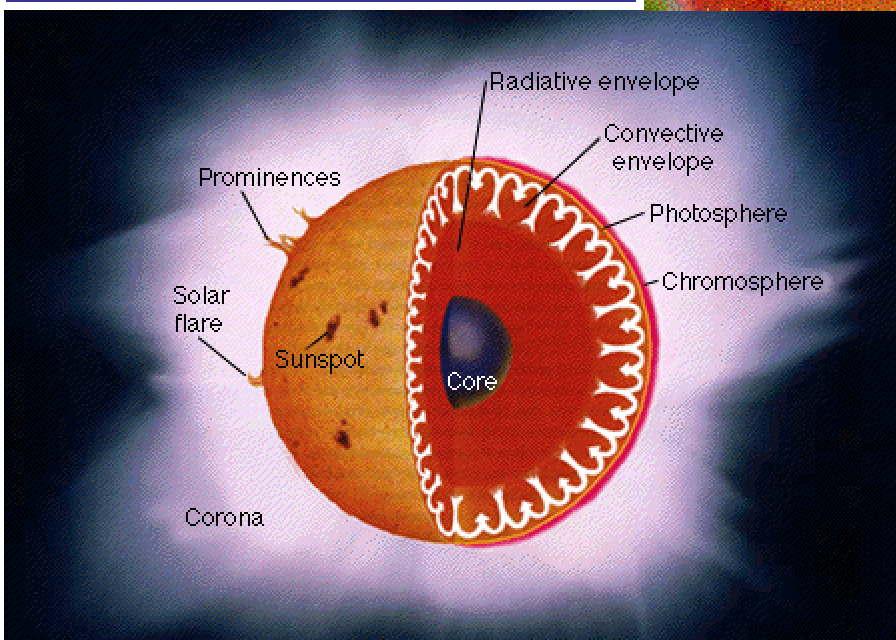
How the Sun Works

The Sun as a star:
1) Core energy production
2) Radiative transfer
3) Convection zone with differential rotation
4) Magnetic fields created via dynamo activity





- The core is the ultimate steady source of energy; however, there is variability in the Sun's energy output due to plasma processes closer to the surface, in the convection zone.
- The Sun's magnetized atmosphere varies on all observable timescales.
- Solar dynamics cause observable effects on Earth and geospace on timescales of seconds to centuries.

Solar Facts and Figures

Observed and inferred from stellar modeling

- Age ~ 4.5 Gyr
- Mass ~ 2 x 10^{30} kg
  Fusion loss rate ~ 4 x 10^{9} kg /s
  Solar wind rate ~ 2 x 10^{9} kg /s
- Radius ~ 700 Mm
  1 AU ~ 1.5 x 10^{5} Mm ~ 215 $R_{\odot}$
  1 AU ~ 8 light-minutes
- Layer thicknesses:
  Core, 175 Mm (25%)
  Radiative zone, 325 Mm (45%)
  Convection zone, 200 Mm (30%)
  Photosphere, 0.5 Mm
  Chromosphere, 2.5 Mm
- Mass fraction below CZ: 98%
- Mean density ~ 1.4 g/cm^3 (H_2O_2)
  Mean composition by number:
  90% H, 10% He, 0.1% other
  Mean composition by mass:
  71% H, 27% He, 3% other
- Conditions at center:
  Temperature, 1.5 x 10^{7} K
  Density, 1.5 x 10^{2} g/cm^3 (gold x 8)
  Mass composition, 34% H,
  64% He, 2% other (C, N, O,…)
- Conditions "at" photosphere:
  Temperature, 5770 K (light bulb)
  Density, 2 x 10^{-7} g /cm^3
  Pressure ~ 0.2 atm
  Gravity ~ 27 gees

Equations of Stellar Structure*

(*i.e., a big, hot ball of gas)

Hydrostatic force balance:

$$\frac{dp(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2} \longleftarrow$$

Mass growth with radius:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

Energy transfer in a radiative zone:

$$\frac{dT(r)}{dr} = -\frac{3\kappa(\rho,T)\rho(r)}{16\sigma T^3(r)} \frac{L(r)}{4\pi r^2} \longleftarrow$$

Energy balance:

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\varepsilon(\rho,T)$$

or in an adiabatic convective zone:

$$\frac{dT(r)}{dr} = -\frac{GM(r)}{r^2} \frac{\delta_p(\rho,T)}{c_p(\rho,T)}$$

(alternate form)

$$\frac{d\ln p(r)}{dr} = \gamma(\rho,T)\frac{d\ln\rho(r)}{dr}$$

Also need: ideal gas equation of state $p = (k_B/\overline{m})\rho T$ and constitutive relations for Rosseland mean opacity κ, nuclear power per gram ε, thermal expansion coefficient $\delta_p(\overline{m})$, specific heat $c_p = \gamma c_v$, etc.

Core Energy Production: pp Chain Reactions

99% of Sun's energy, 2% of it in neutrinos

(pp) $^{1}H + {}^{1}H \rightarrow {}^{2}D + e^{+} + \nu_e$

$e^{+} + e^{-} \rightarrow 2\gamma$

(pep) $^{1}H + e^{-} + {}^{1}H \rightarrow {}^{2}D + \nu_e$

(.0025)

$^{2}D + {}^{1}H \rightarrow {}^{3}He + \gamma$

pp I: .87

(.13)

$^{3}He + {}^{3}He \rightarrow {}^{4}He + 2\,{}^{1}H$

(pp) $2\,{}^{1}H \rightarrow {}^{2}D + e^{+} + \nu_e$

$^{2}D + {}^{1}H \rightarrow {}^{3}He + \gamma$

$e^{+} + e^{-} \rightarrow 2\gamma$

$^{3}He + {}^{4}He \rightarrow {}^{7}Be + \gamma$

pp II: .129

pp III: .001

$^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_e$

$^{7}Be + {}^{1}H \rightarrow {}^{8}B + \gamma$

pp IV: <10^{-6}

(Hep) $^{3}He + {}^{1}H \rightarrow {}^{4}He + e^{+} + \nu_e$

$e^{+} + e^{-} \rightarrow 2\gamma$

$^{7}Li + {}^{1}H \rightarrow {}^{4}He + {}^{4}He$

$^{8}B \rightarrow {}^{8}Be^{*} + e^{+} + \nu_e$

$e^{+} + e^{-} \rightarrow 2\gamma$

$^{8}Be^{*} \rightarrow {}^{4}He + {}^{4}He$

Net: $4\,{}^{1}H + 2e^{-} \rightarrow {}^{4}He + 2\,\nu_e$
+ 26.731 MeV (2.8% of proton mass)

Why is it Good that the pp Chain Starts Slowly?

$$\text{(pp)}\quad {}^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{D} + e^{+} + \nu_e$$

- The pp reaction is actually a combination of two events:
  - Two protons momentarily fuse into a highly unstable helium-2 nucleus
  - The ^{2}He undergoes beta decay before it dissociates!
  - This mechanism was first proposed by Hans Bethe in 1939
- The process is therefore dependent on the weak nuclear interaction
  - Beta decay is relatively slow ("beta plus" = positron emission)
  - This type of event is accordingly rare; it's the rate-limiting step
- Why is this good?
  - **If it went faster, the Sun would have exhausted all its hydrogen long ago**
- The Hep reaction is similarly rare, as it depends on the beta decay of ^{4}Li :

$$\text{(Hep)}\quad {}^{3}\mathrm{He} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + e^{+} + \nu_e$$

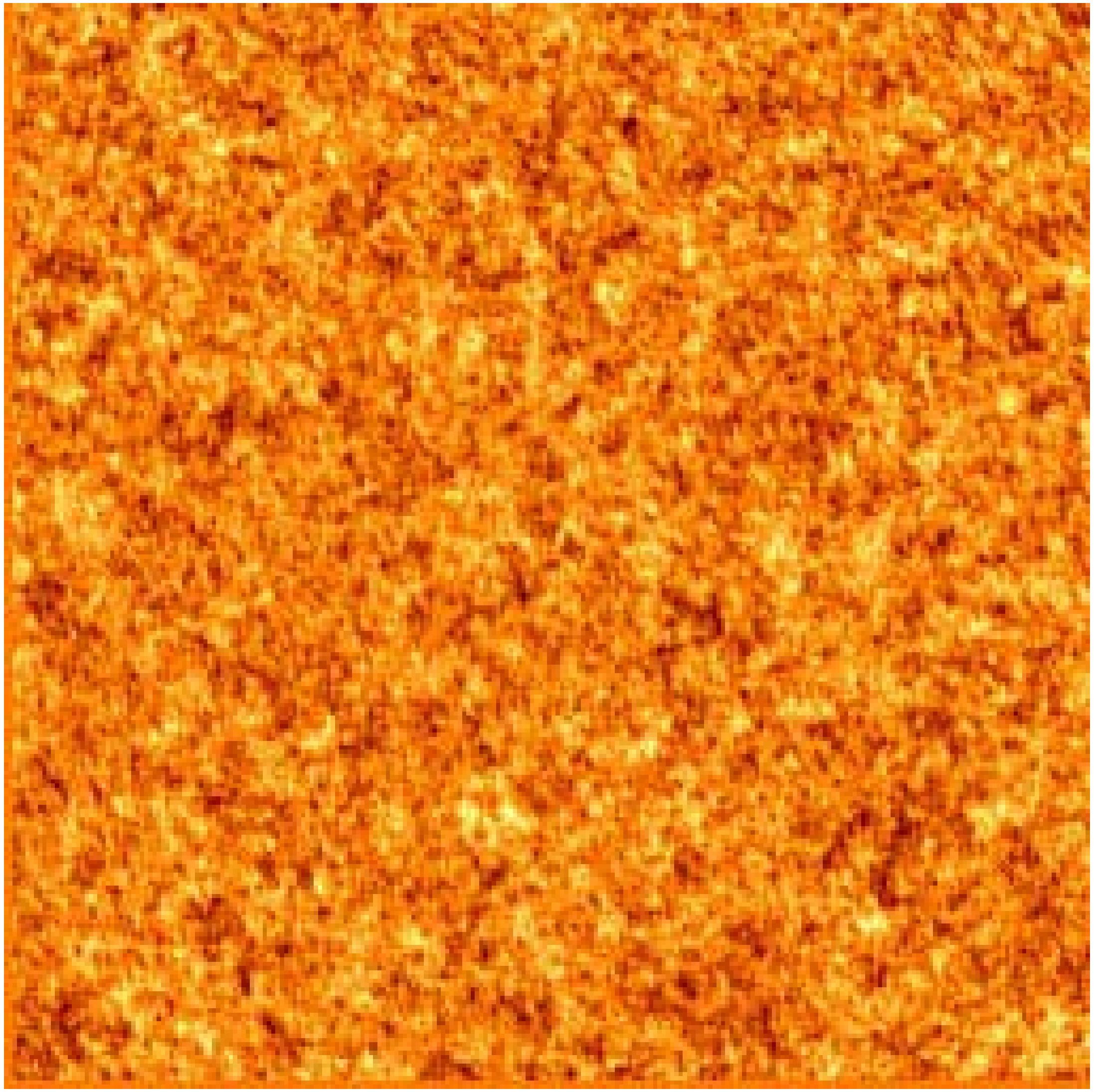
Refining the Stellar Model through Observations

How we study various regions and properties of the Sun:

- **Nuclear fusion:** *neutrino telescopes*
- **Core:** *low-degree p-modes (pressure/acoustic modes) from helioseismology*
- **Radiative zone:** *low-degree p-modes*
- **Convective zone:** *moderate-degree p-modes*
- **Sub-surface zone:** *high-degree p-modes*
- **Surface conditions, motion, and magnetic field:** *visible light images; tracking of features or structures; imaging of Doppler and Zeeman shifts*
- **Chromosphere:** *UV/EUV Imaging and spectroscopy (higher temperature)*
- **Corona:** *EUV/X-ray imaging and spectroscopy, in-situ particle and field measurements*

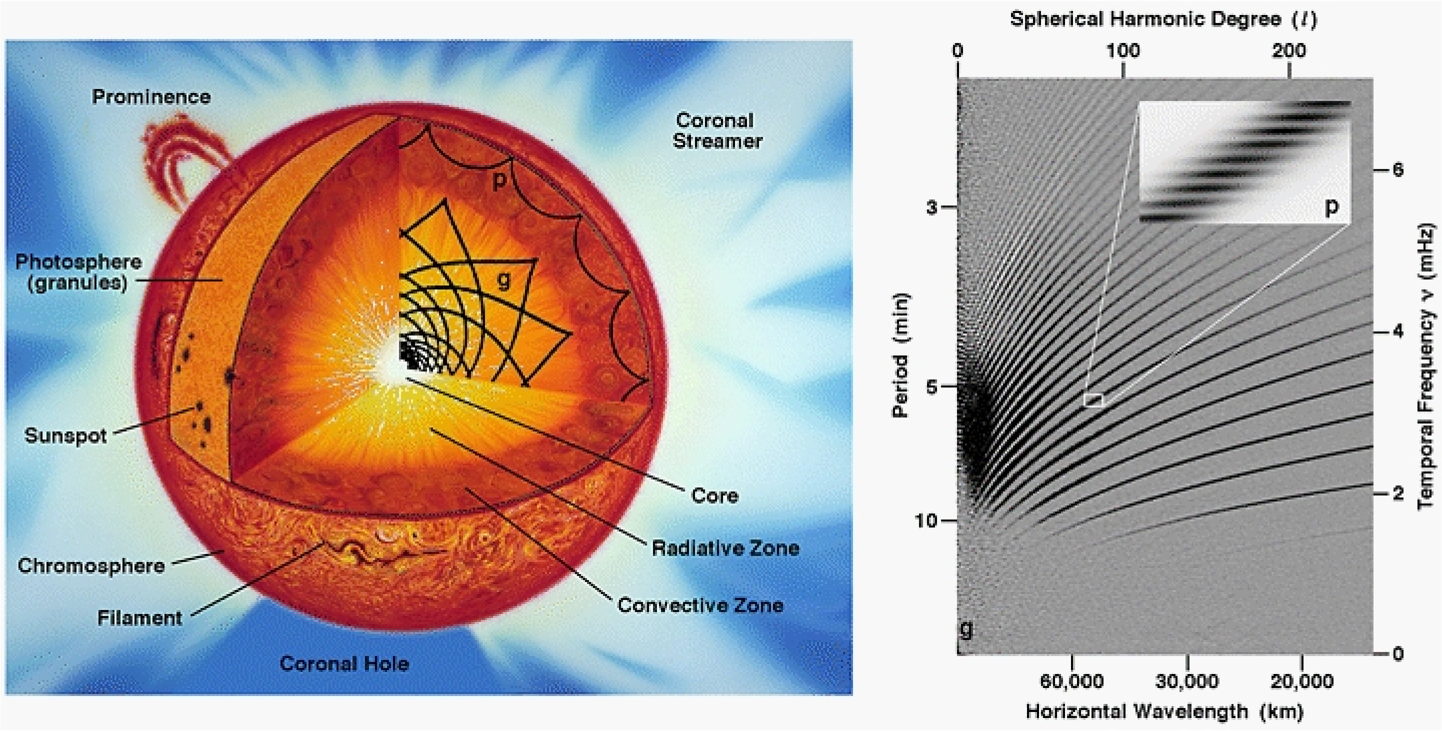
Doppler Imaging Reveals Photospheric Oscillations

They dominate over the convection “granules” seen in white light



Helioseismology

Study of the Sun’s normal modes of vibration (spherical harmonics)

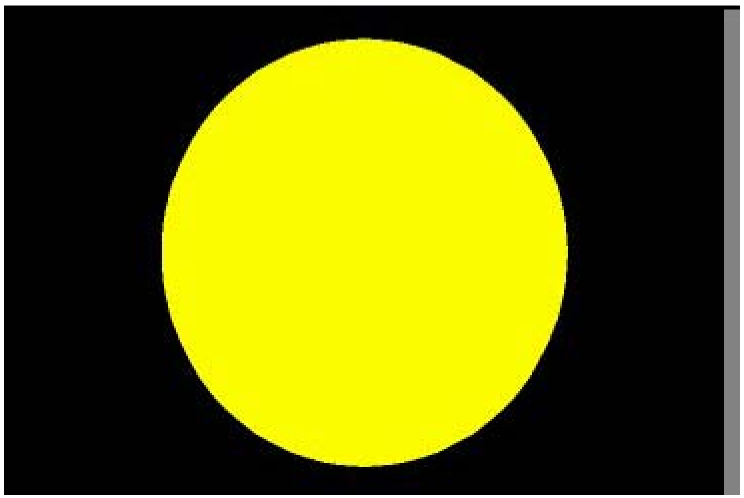


http://sohowww.nascom.nasa.gov/gallery

Power spectrum has ridges for all radial degrees *n*

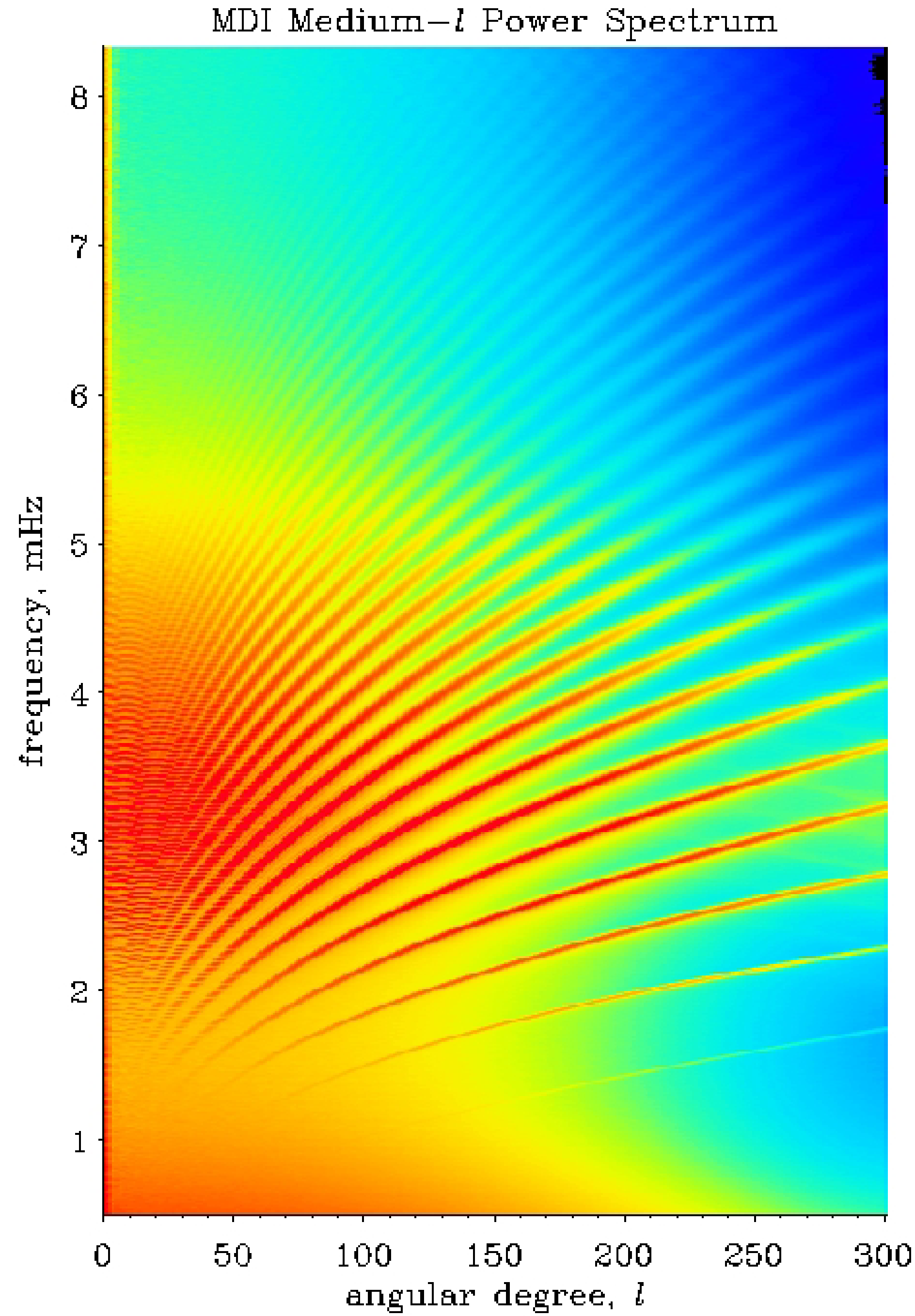
Power Spectrum of 5-Minute Oscillations

Color-coded: red highest



Movie shows that longer wavelengths probe deeper

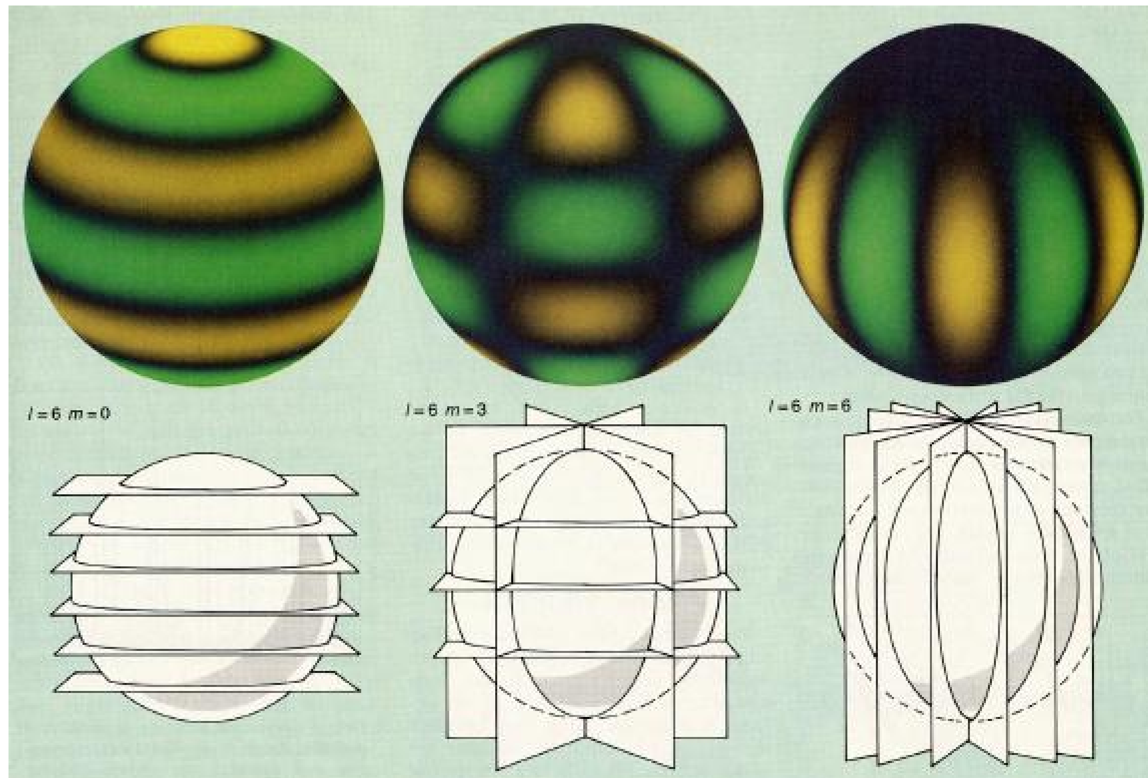
http://sdo.gsfc.nasa.gov



What *p*-Modes Look Like

Illustrations of spherical harmonics

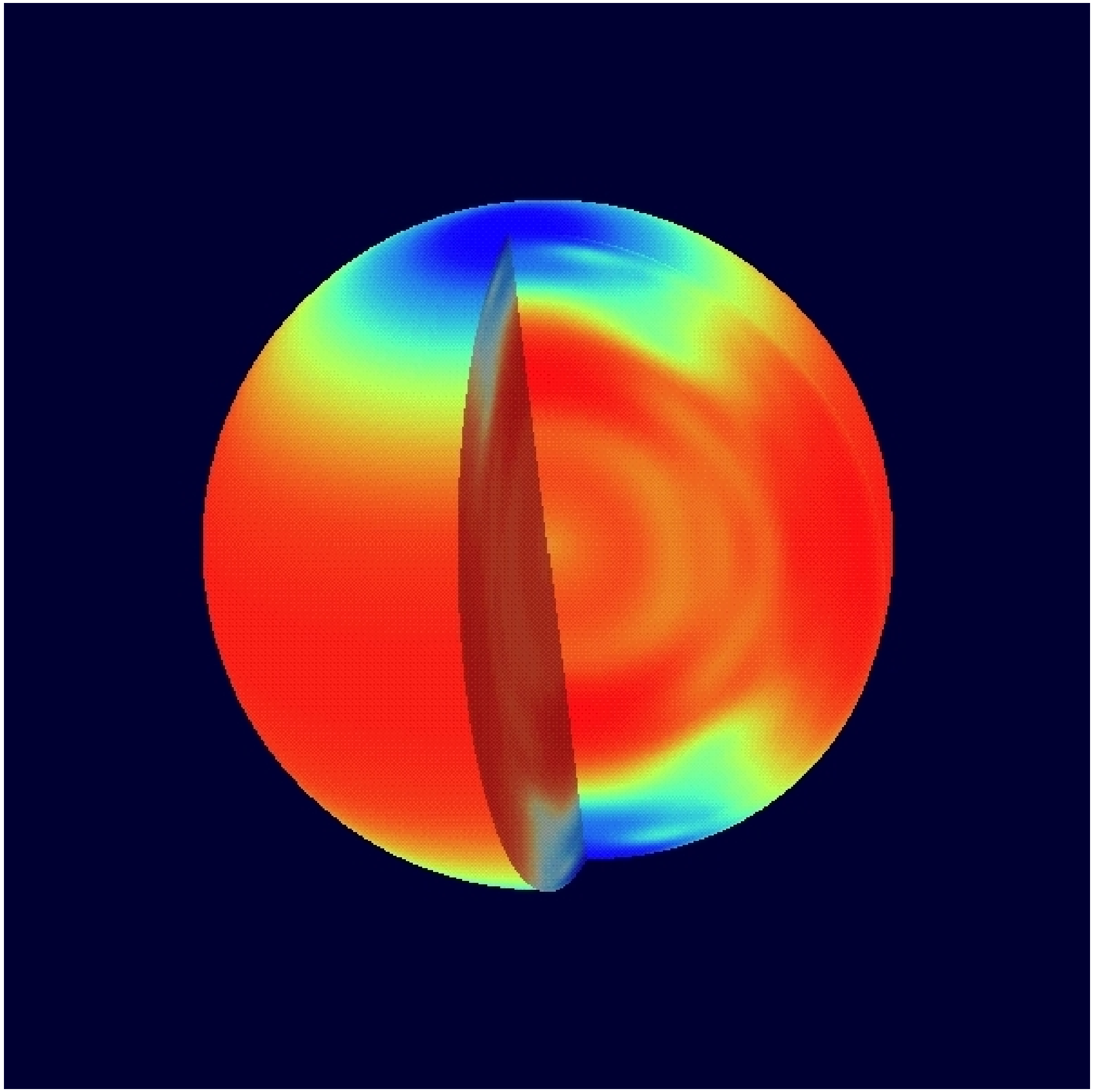
A *p*-mode has 3 "quantum numbers": n = number of radial nodes, l = number of nodal planes cutting the surface, m = number of nodal planes cutting the equator



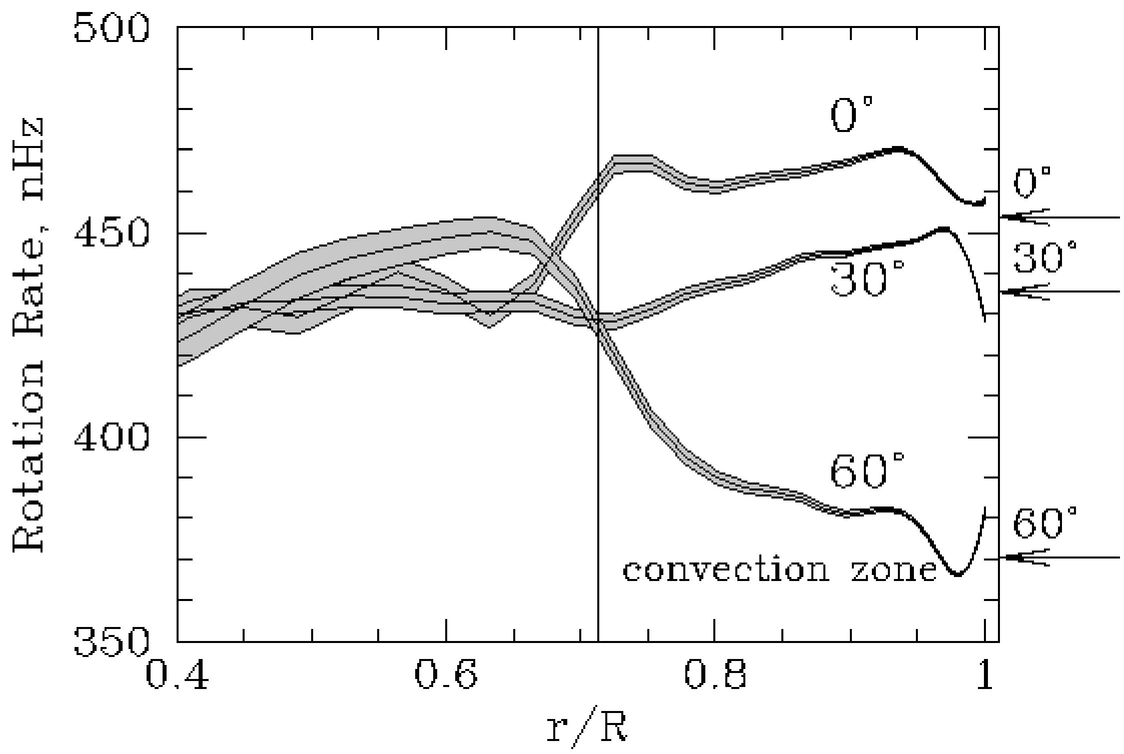
Rotation implies a Doppler shift that breaks the +/- degeneracy in m, so the standing wave precesses

http://solarscience.msfc.nasa.gov/Helioseismology.shtml & http://www.oca.eu/grec/astrosismologie.html

Portrait of Differential Rotation from Helioseismology



Within the Convection Zone, Rotation Rate Varies with Latitude



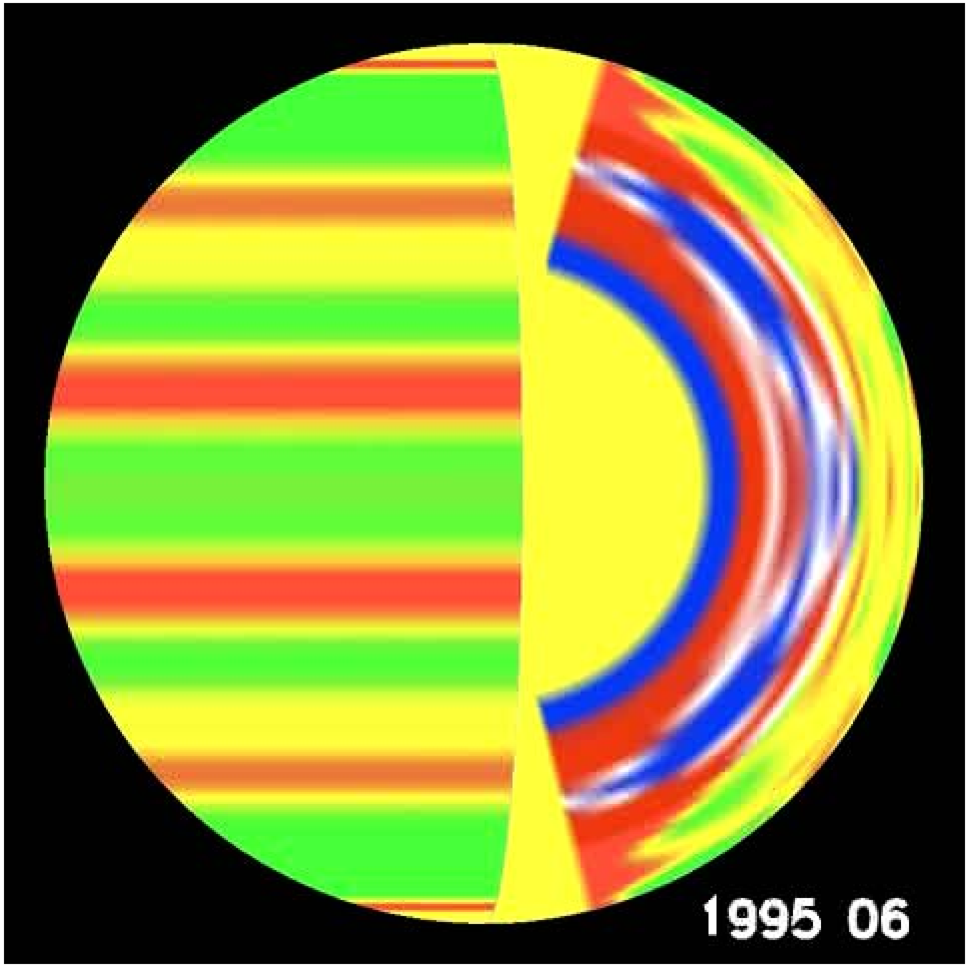
- Average rotation speed as a function of depth and latitude can be inferred from the Doppler splitting of peaks in the power spectrum
- Eastward- and westward-traveling waves have different resonant frequencies

How Differential Rotation Might Drive the Dynamo

“Frozen-in” flux is wound around the base of the convection zone

http://sdo.gsfc.nasa.gov/epo/educators/presentations/presentations2006_07_esss.php

“Jet Streams” and Pulses in the Sun’s Differential Rotation

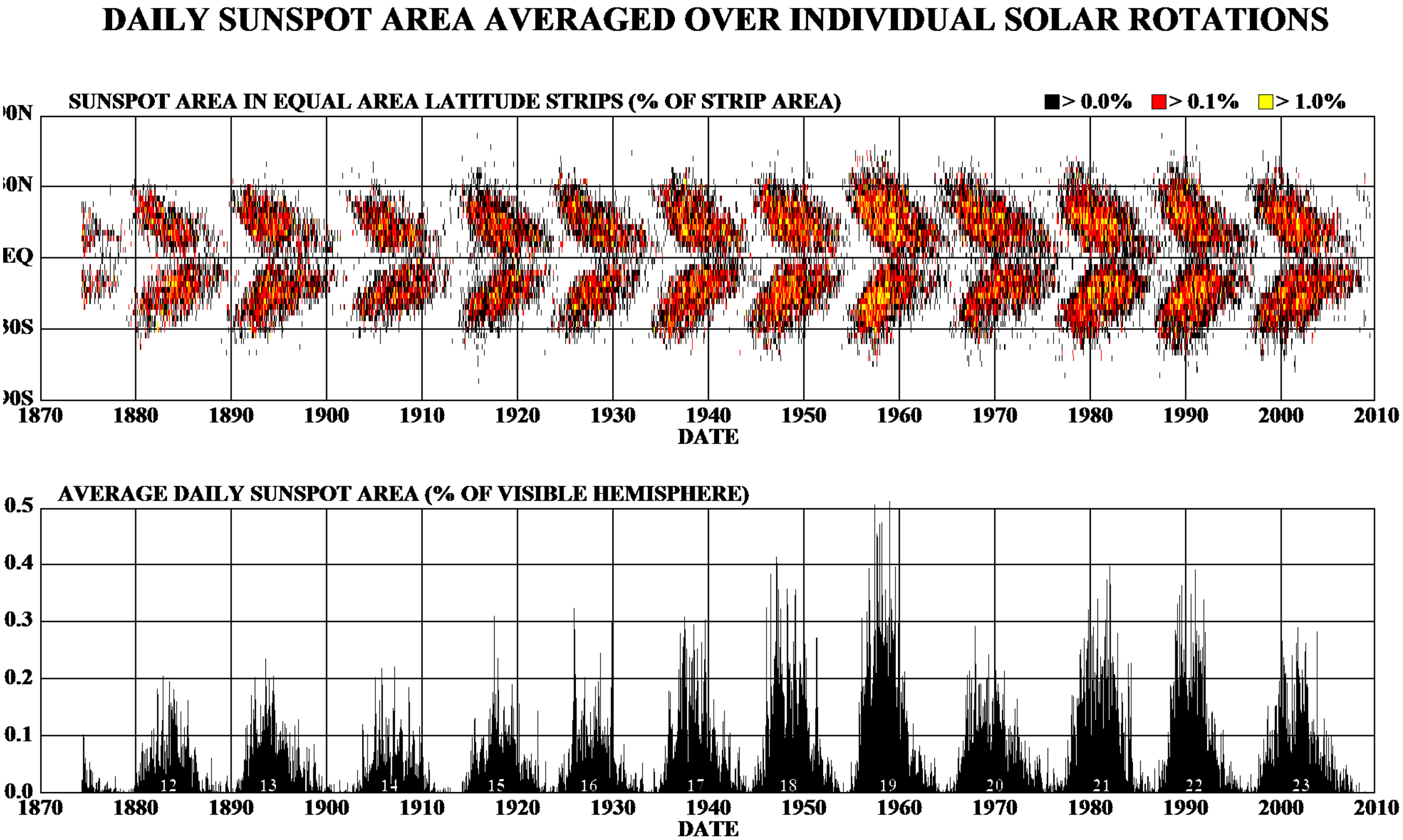


http://spd.boulder.swri.edu/solar_mystery

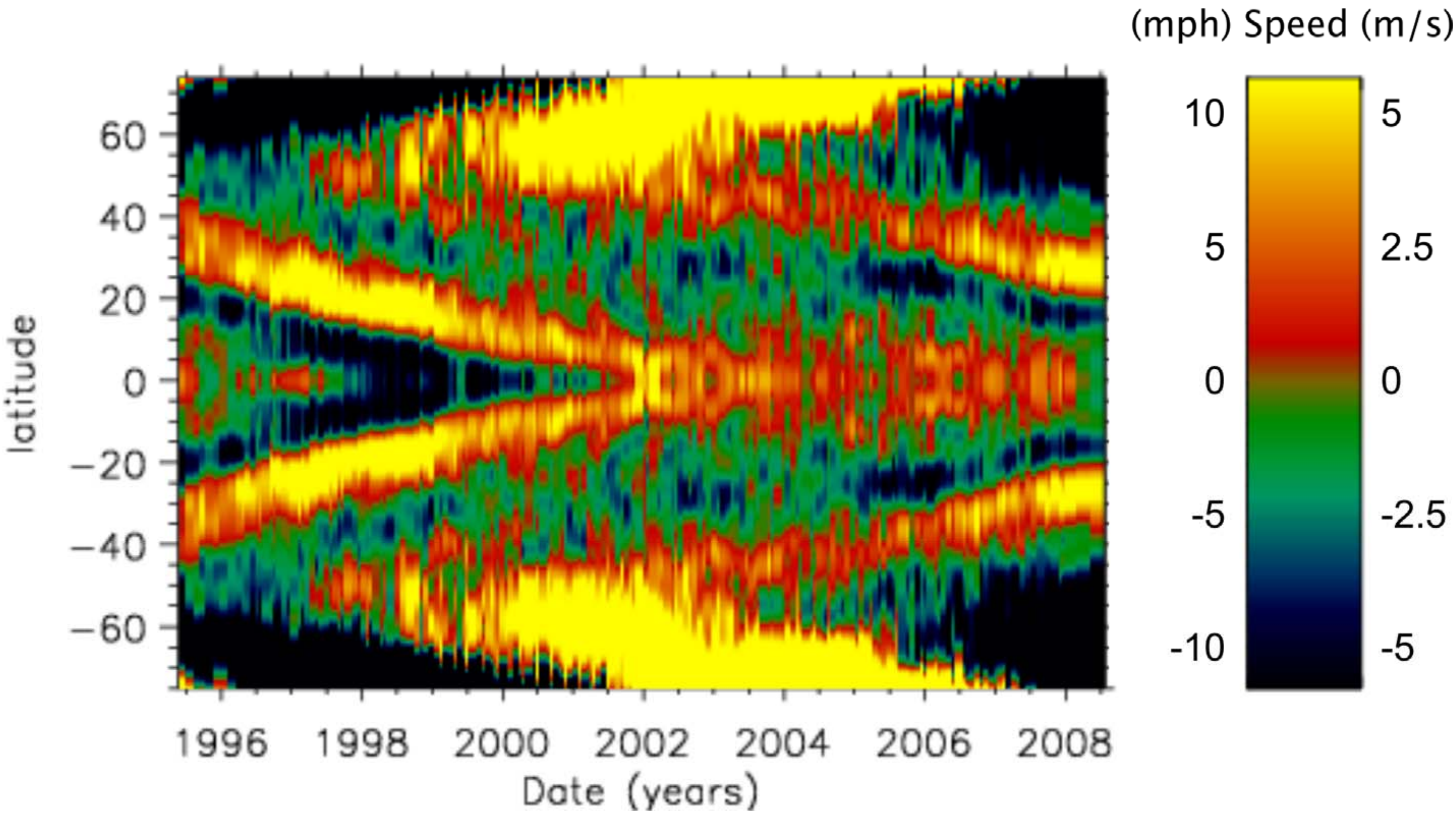
- The movie is based on 13+ years of helioseismology from SOHO/MDI and GONG. Rotation *variations* are color coded: blue/red is slow/fast.
- Red bands in the outer third of the Sun migrate slowly down from each pole toward the equator. Claim: these torsional oscillations or “jet streams” are linked with sunspot emergence and the solar cycle.
- A mystery: rotation rates at the base of the convection zone, the level of the suspected dynamo, change markedly or “pulse” over 6–7 months.

The Solar Cycle: “Butterfly Diagram”

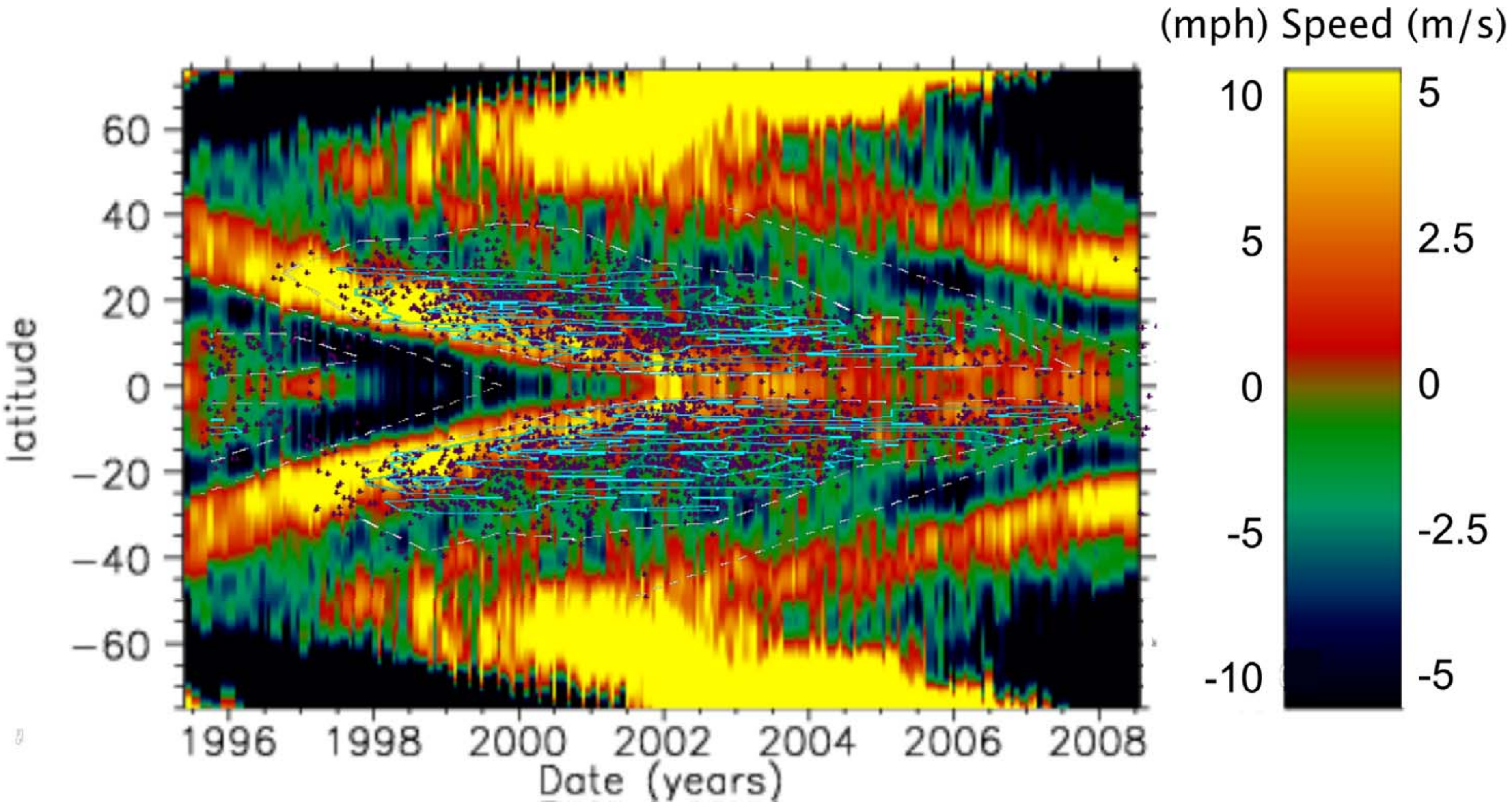
Latitudes of sunspot appearance vs. time



7 Megameters (1%) Down:
Torsional Oscillation or "Jet Stream"

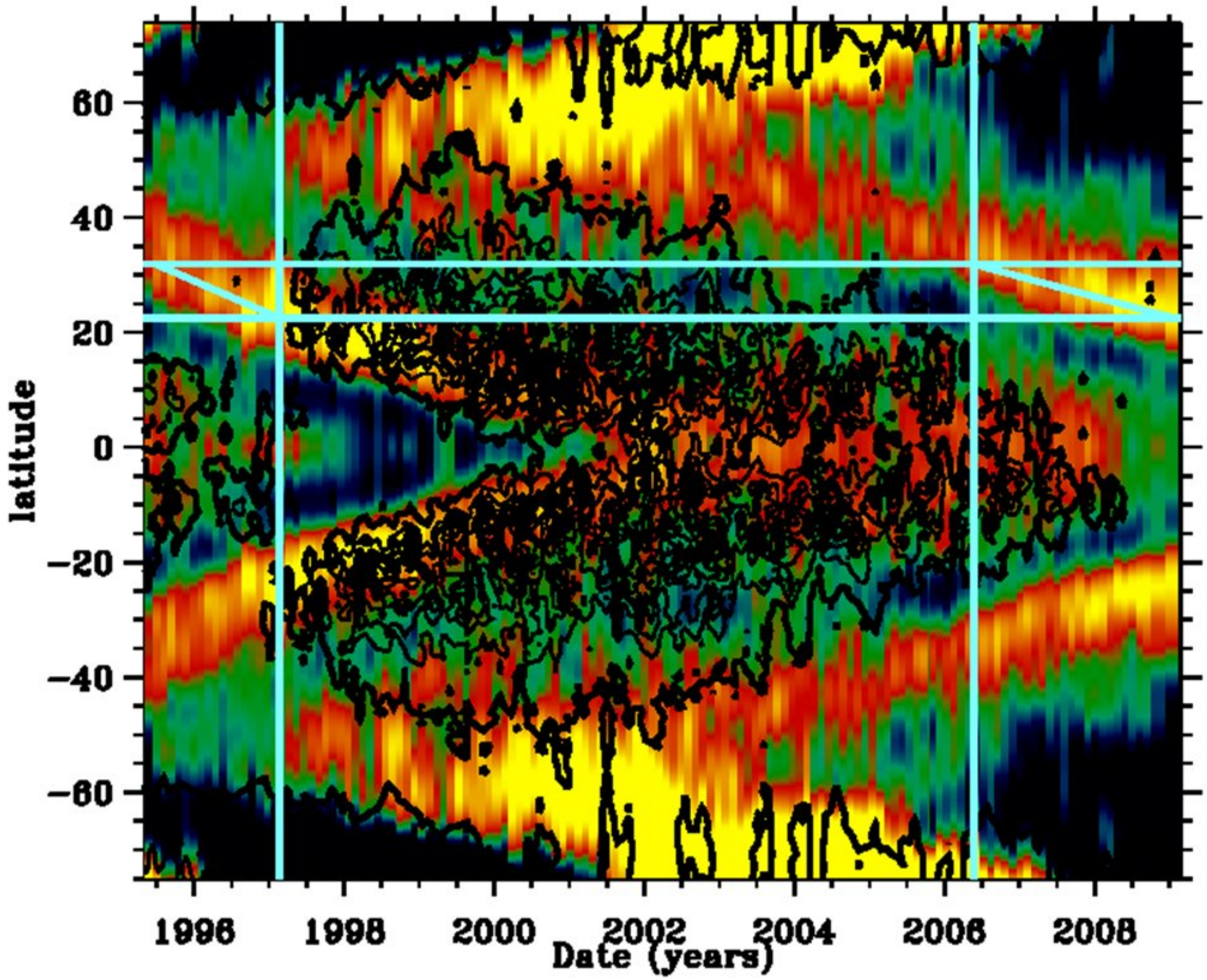


Overlay of Jet Streams with the Butterfly Diagram



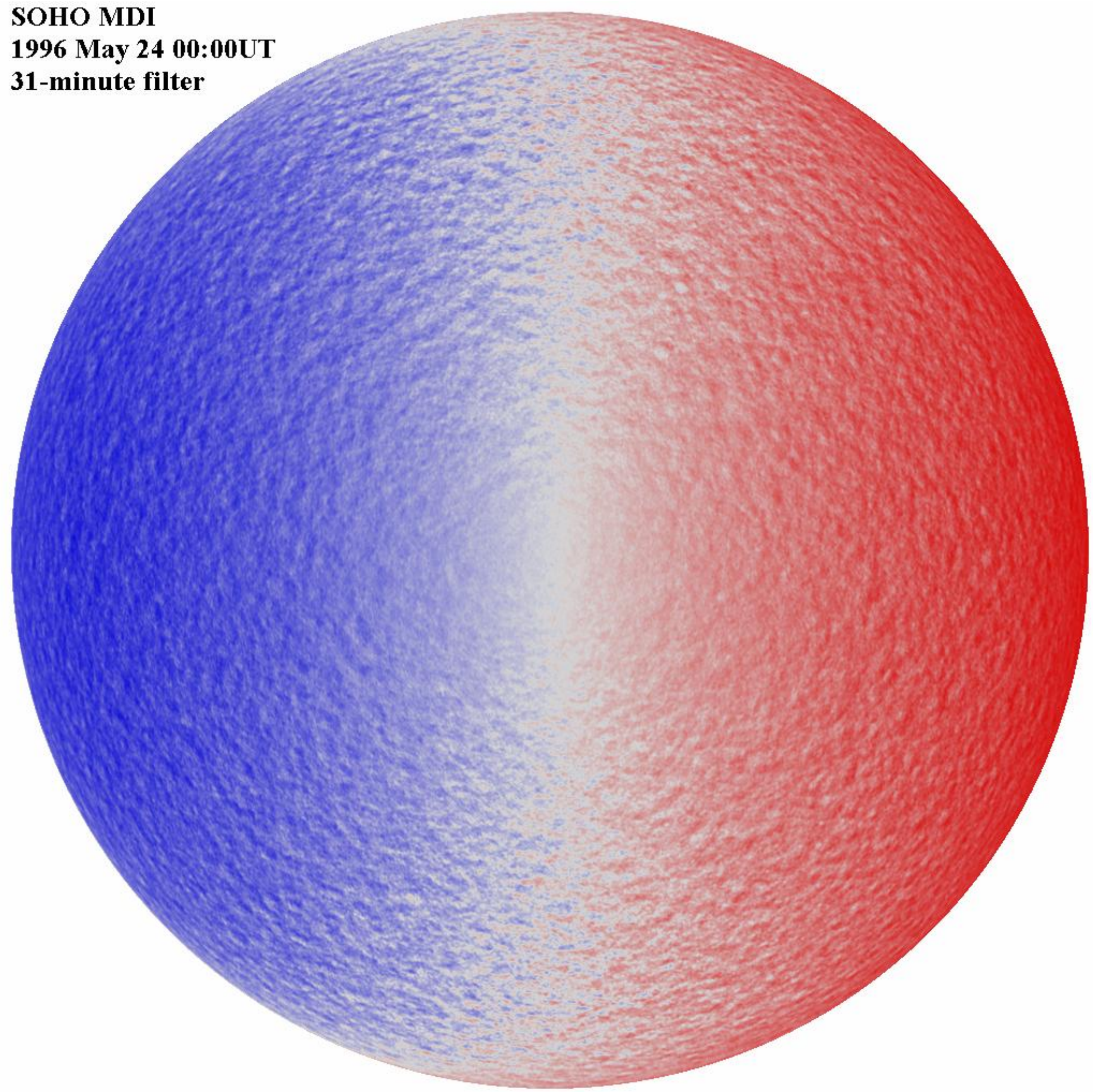
Jet Migration is Slower in The Present Solar Cycle…

Does it explain the persistence of the previous solar minimum?



http://science.nasa.gov/media/medialibrary/2009/06/17/17jun_jetstream_resources/sonogram.jpg

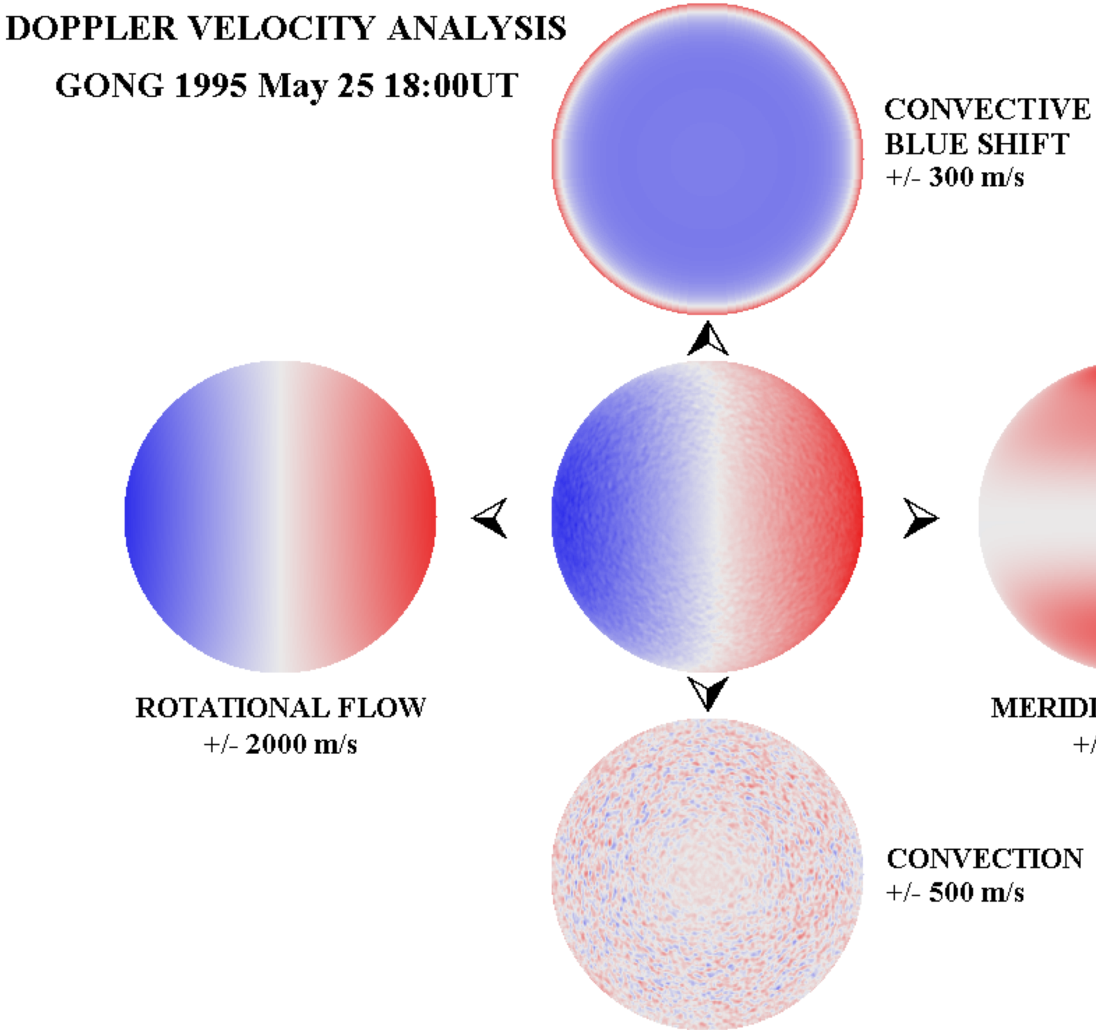
Full-Disk Dopplergram Showing Steady Surface Flows



Filtered to remove *p*-modes

http://solarscience.msfc.nasa.gov/

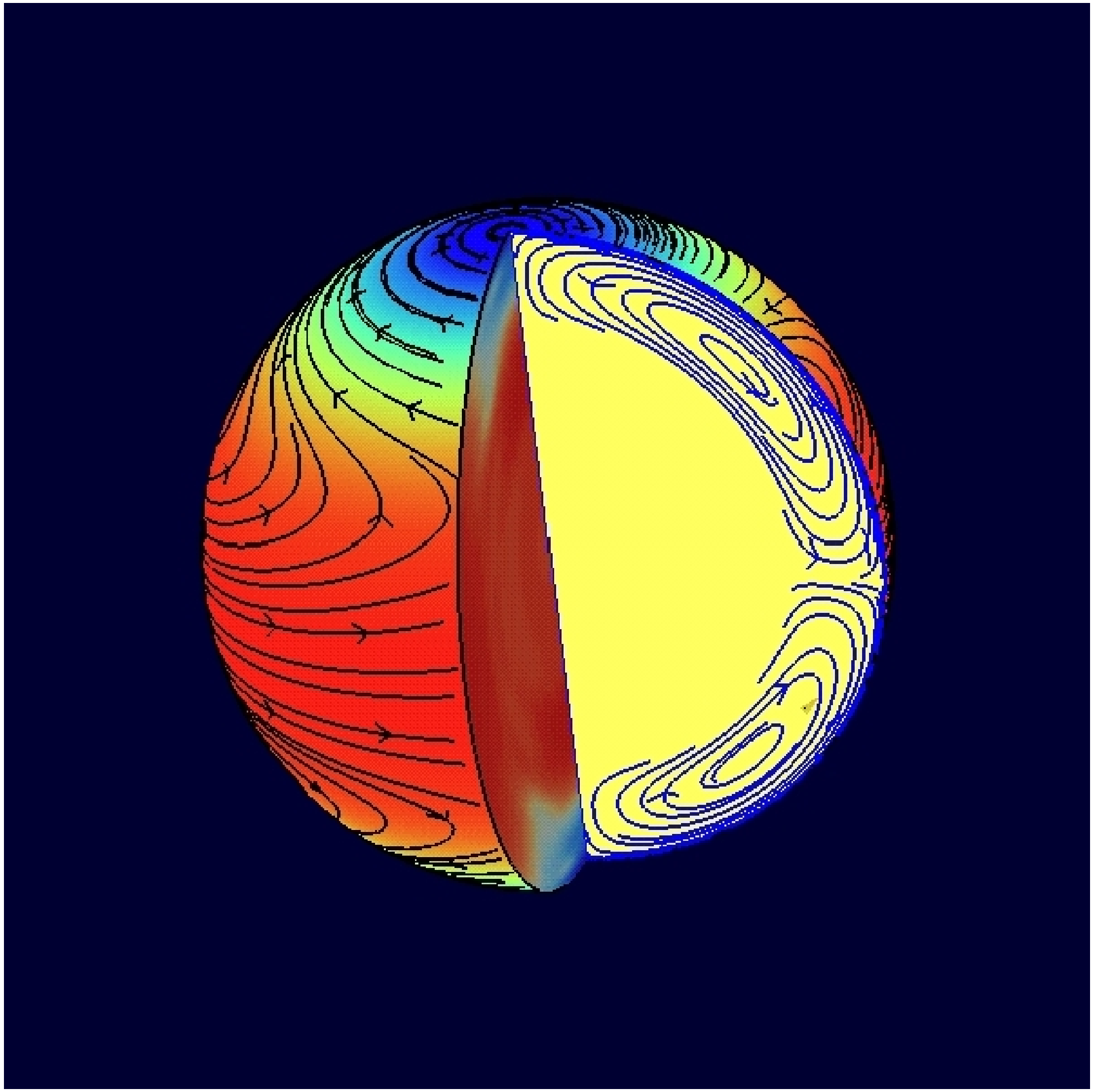
Analysis: Separated Components of the Flow



Regions of upflow are broader—they occupy more area

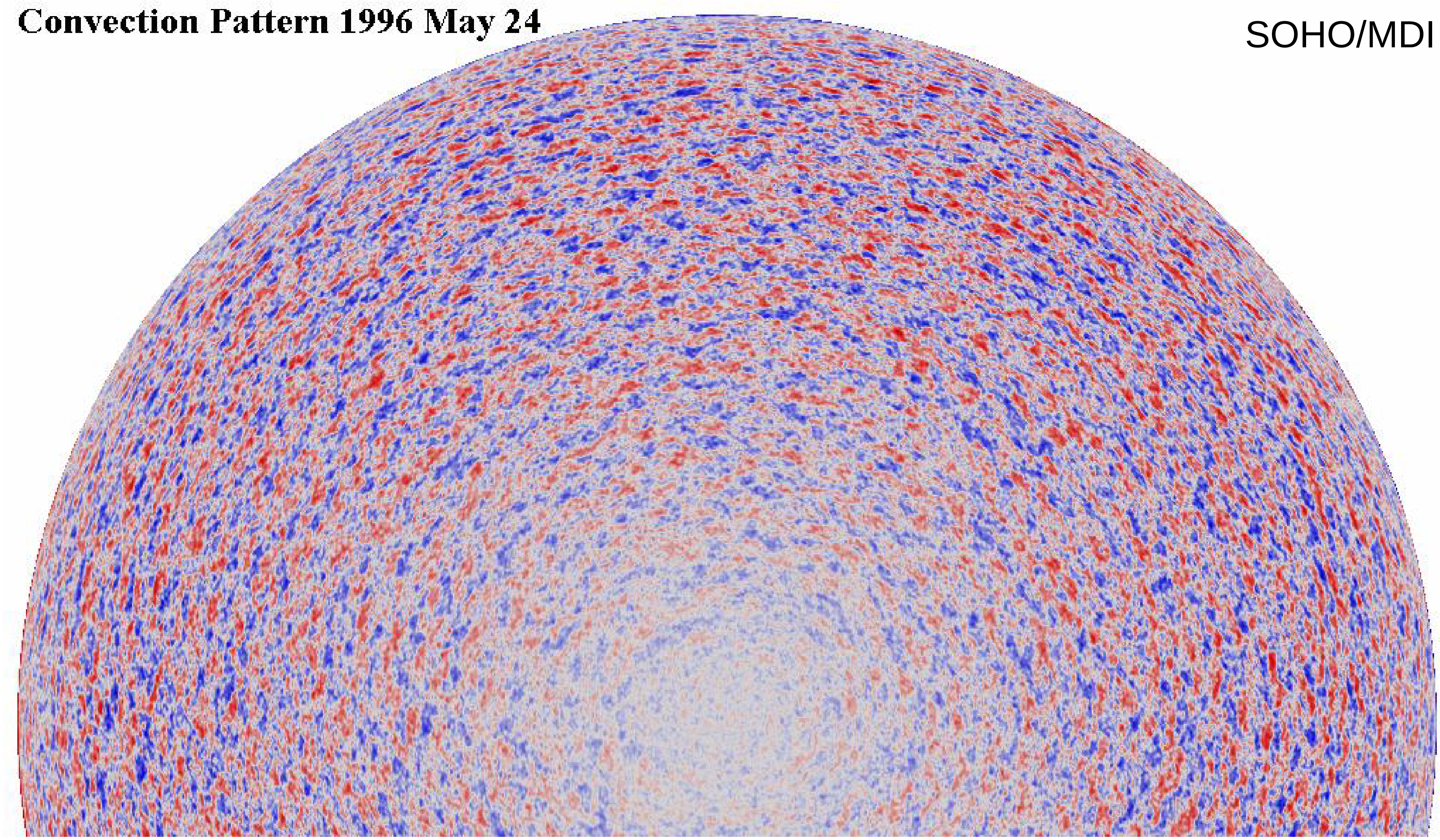
NASA/MSFC Hathaway

Hypothetical Streamlines Including Meridional Flow



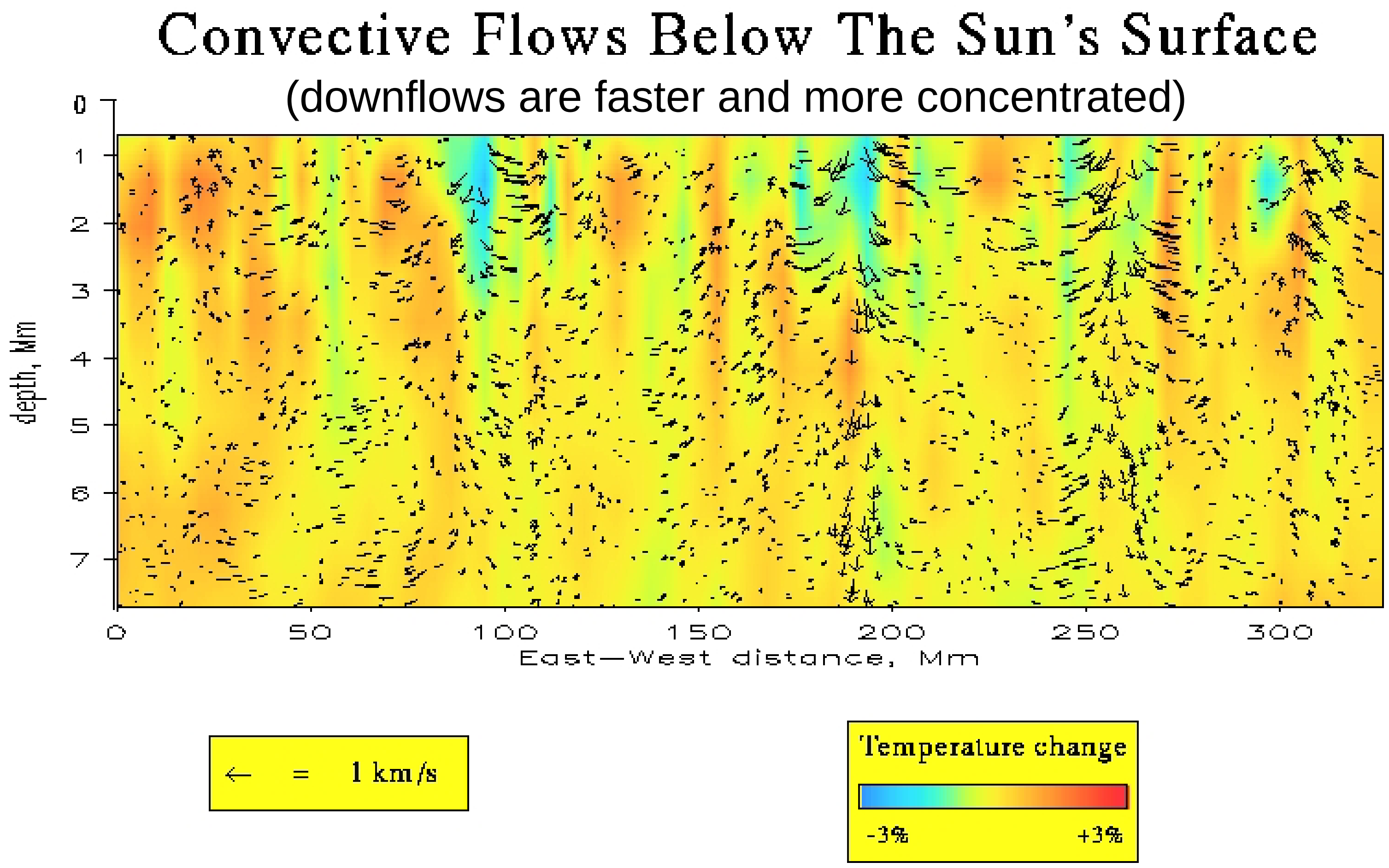
Supergranulation

Detected through the horizontal component of the flow



SOHO/MDI

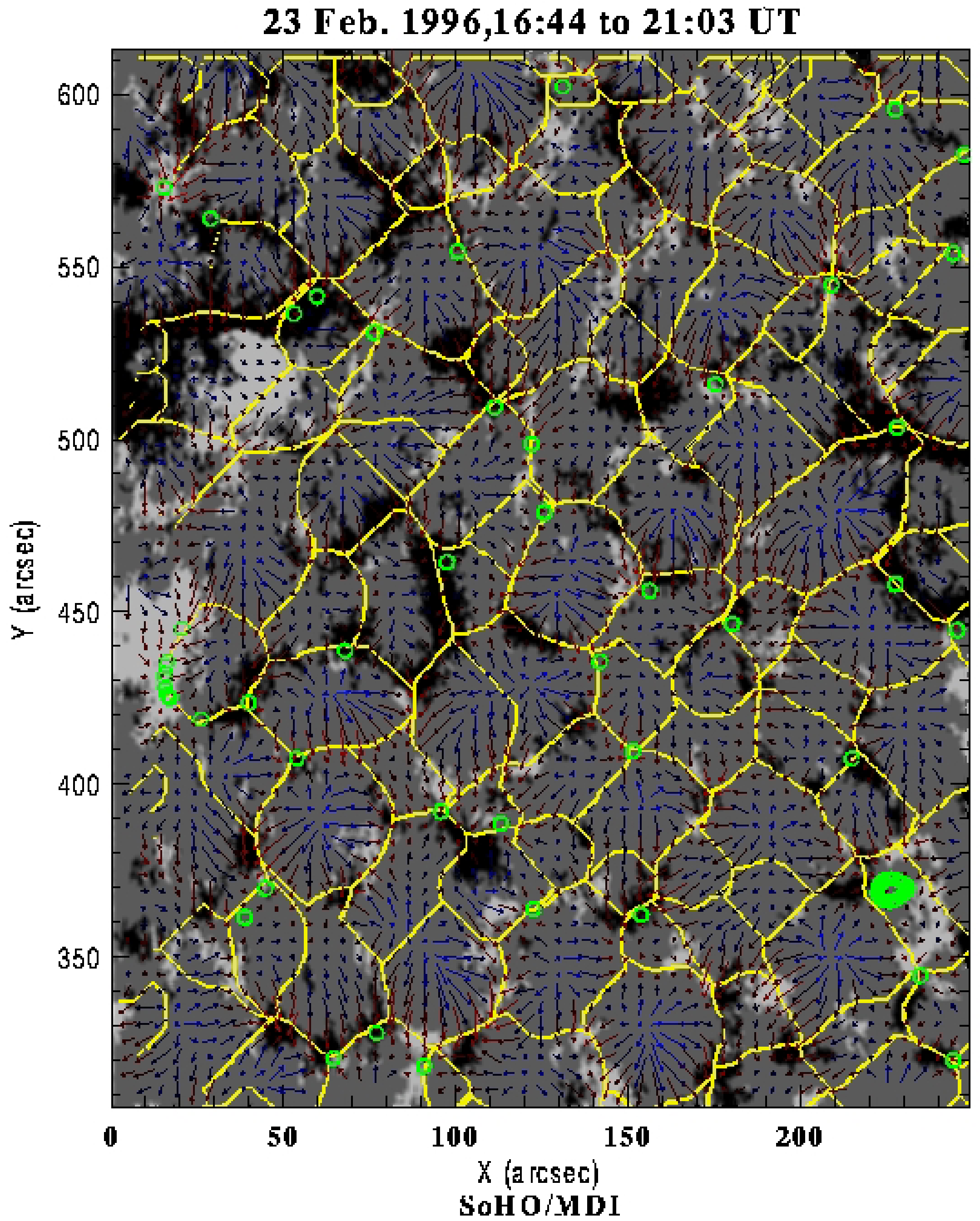
Local Time-Distance Helioseismology Also Reveals…



A vertical cut through the outer 1% of the sun showing flows and temperature variations inferred by helioseismic tomography.

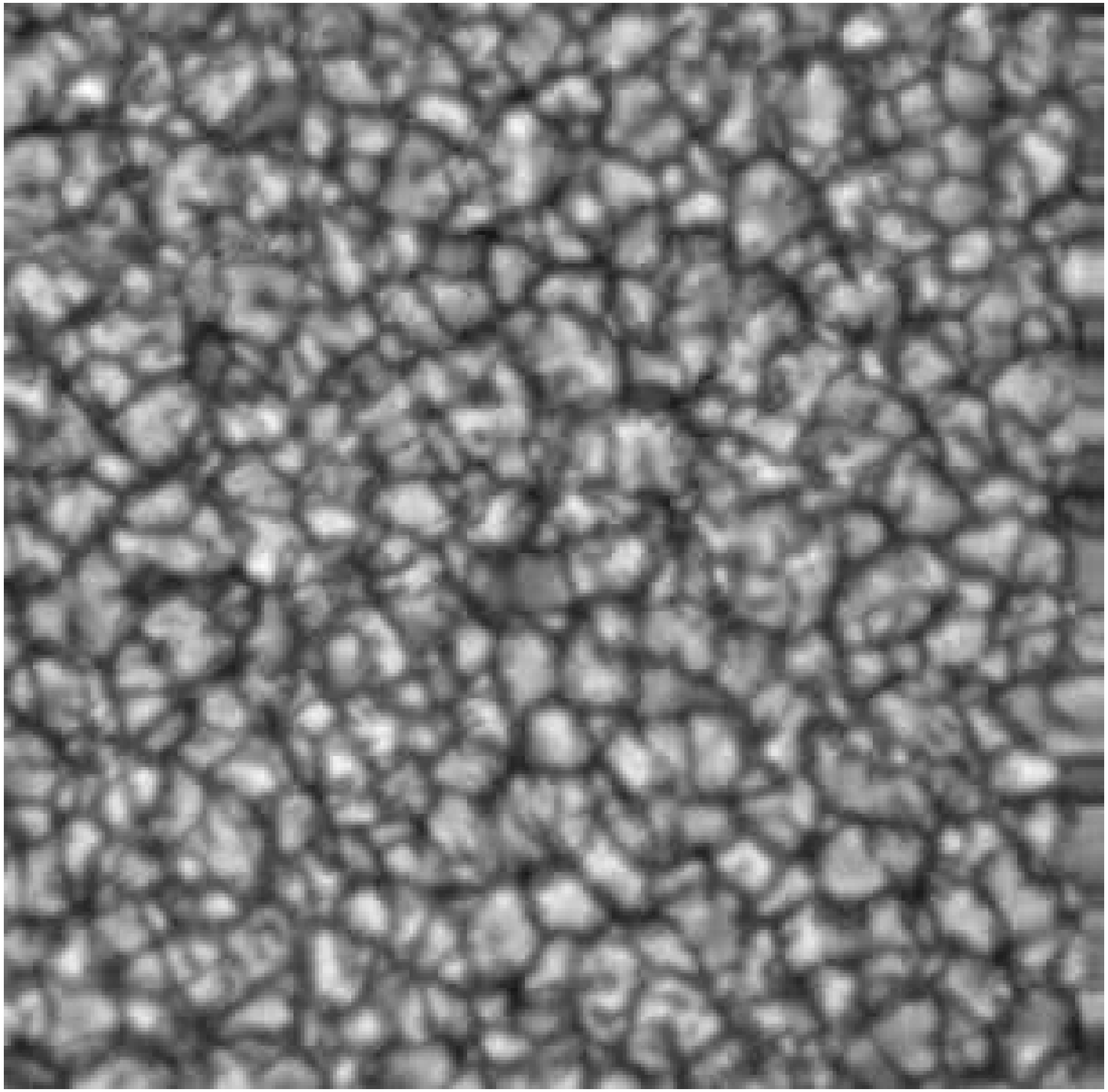
Magnetic Flux at the Supergranule Boundaries

- Magnetic flux is observed to congregate at the edges of supergranules
- Lines and points of convergence of the horizontal flow are drawn in yellow and green
- Lines are inferred to be regions of downflow
- Opposite magnetic field polarities are depicted in black and white
- Analogy: like corks floating in a pot of boiling water



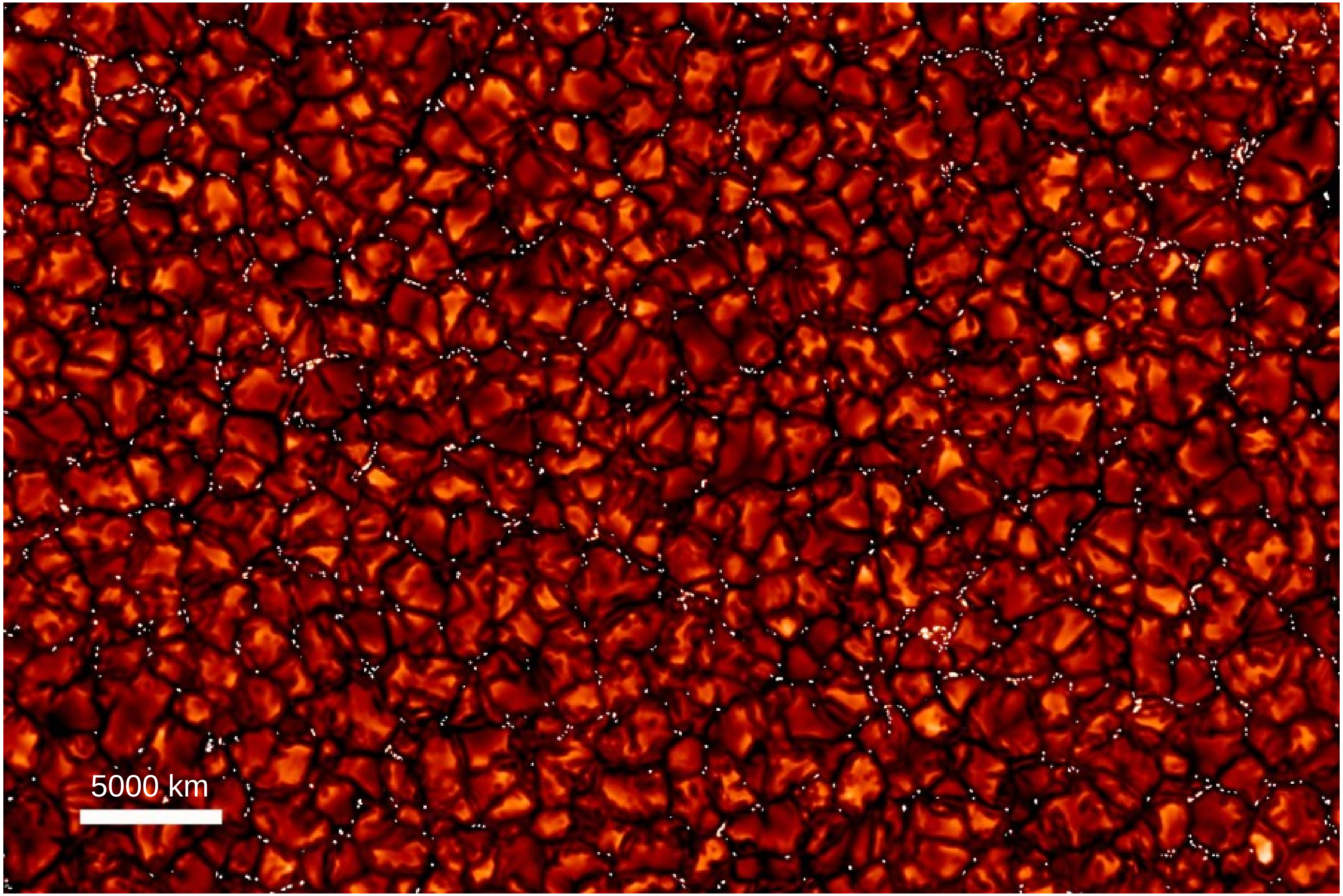
Granulation

This type of solar convection was first discovered in high-resolution white-light images over 100 years ago; now Doppler images tell us the line-of-sight speed



Granulation Also Drives Magnetic Fields to Downflows

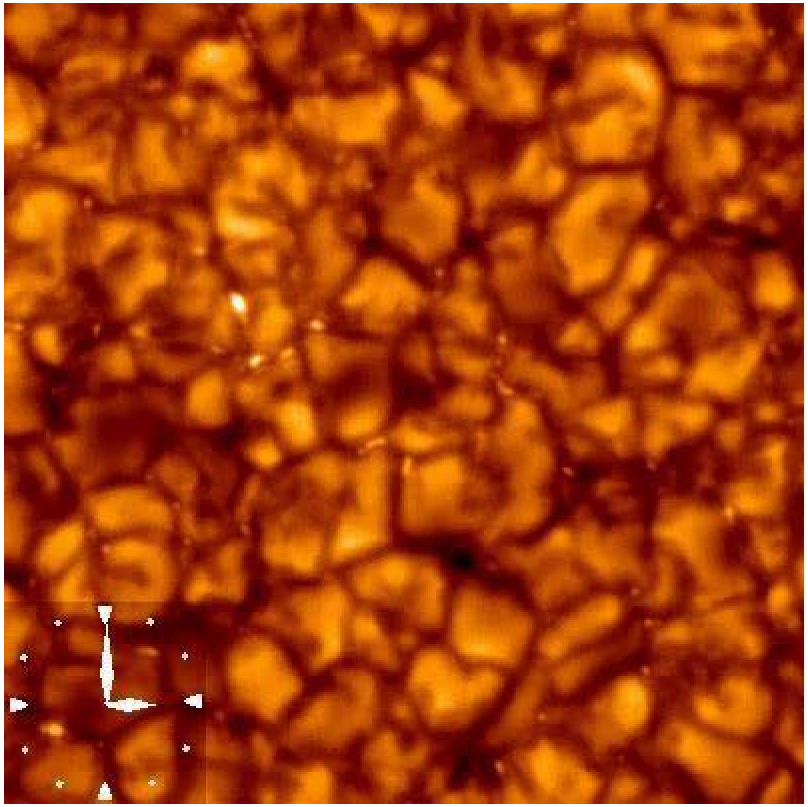
X ray bright points (XBP, in white) are associated with strong fields

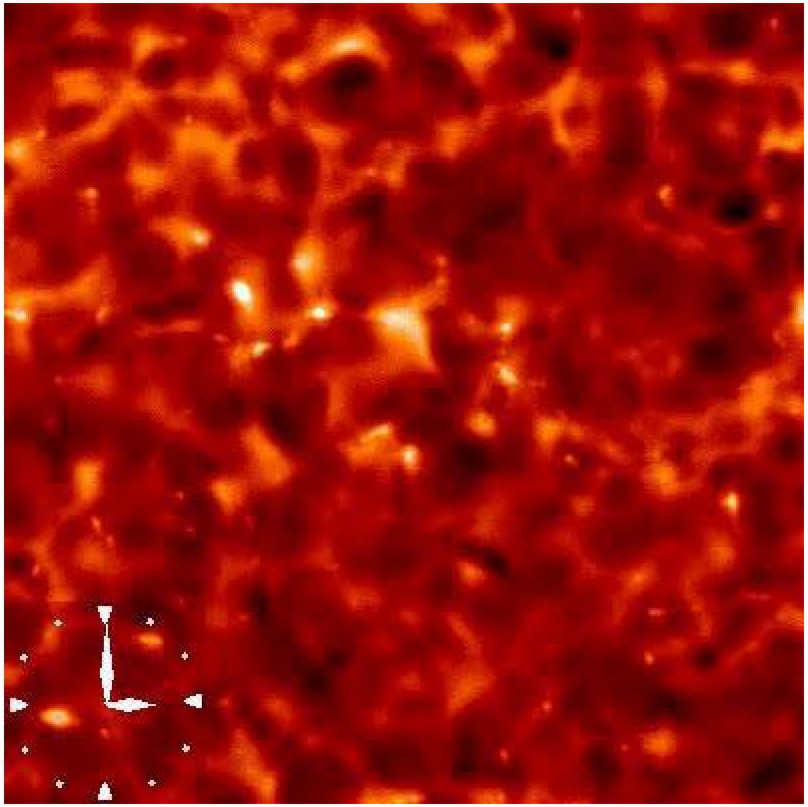


http://antwrp.gsfc.nasa.gov/apod/ap100416.html

Dynamics of Magnetic Fields at Granule Boundaries

Yohkoh observations of the CH G band (left) and Ca II H line (right)

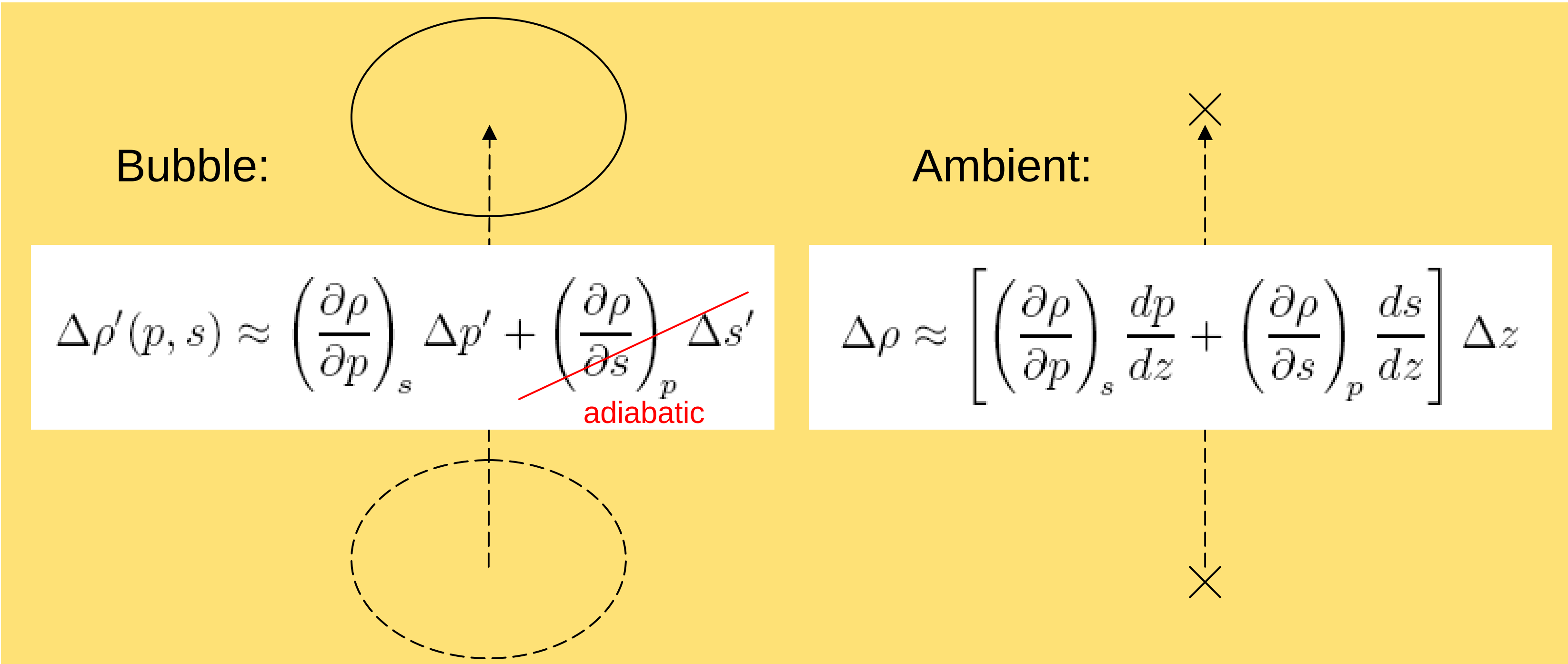




http://solar-b.nao.ac.jp/news_e/20061127_press_e

Instability to Convection

The Schwarzschild criterion



Bubble:

$$\Delta\rho'(p,s) \approx \left(\frac{\partial\rho}{\partial p}\right)_s \Delta p' + \cancel{\left(\frac{\partial\rho}{\partial s}\right)_p \Delta s'}$$

adiabatic

Ambient:

$$\Delta\rho \approx \left[\left(\frac{\partial\rho}{\partial p}\right)_s \frac{dp}{dz} + \left(\frac{\partial\rho}{\partial s}\right)_p \frac{ds}{dz}\right] \Delta z$$

Difference: $$\Delta\rho' - \Delta\rho \approx \left(\frac{\partial\rho}{\partial p}\right)_s \cancel{\left[\Delta p' - \frac{dp}{dz}\Delta z\right]} - \left(\frac{\partial\rho}{\partial s}\right)_p \frac{ds}{dz}\Delta z$$

pressure equil.

$$\boxed{-\left(\frac{\partial\rho}{\partial s}\right)_p \frac{ds}{dz}\Delta z = -\frac{T}{c_p}\left(\frac{\partial\rho}{\partial T}\right)_p \frac{ds}{dz}\Delta z = \frac{\rho\delta_p}{c_p} \cdot \frac{ds}{dz}\Delta z < 0}$$

for instability

How Unstable Does the Convection Zone Need to Be?

1) Heat per unit mass released by a bubble rising (or falling) over a "mixing length":

$$T\Delta s' - T\Delta s = -T\ell\frac{ds}{dz}$$

2) Convective heat flux due to the average speed of all the rising (and falling) bubbles:

$$F_{\text{conv}} = \tfrac{1}{2}\rho V T\ell\frac{ds}{dz}$$

3) Take the local mixing length to be proportional to *H*…

$$\ell = aH$$

4) …where the local pressure scale height *H* is defined as:

$$H \equiv -\left(\frac{d\ln p}{dz}\right)^{-1} \approx \frac{p}{\rho g} = \frac{\alpha_T}{\gamma g}{c_s}^2$$

The above assumes zero-order hydrostatic balance and defines c_s as:

$$c_s^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_s = -\left(\frac{\partial s}{\partial \rho}\right)_p\left(\frac{\partial p}{\partial s}\right)_\rho = -\frac{c_p}{c_v}\left(\frac{\partial T}{\partial \rho}\right)_p\left(\frac{\partial p}{\partial T}\right)_\rho = \gamma\left(\frac{\partial p}{\partial \rho}\right)_T = \frac{\gamma}{\alpha_T}\left(\frac{p}{\rho}\right)$$

Estimating the Mach Number of the Convection

1) Kinetic energy of a bubble due to buoyancy acting over half a "mixing length":

$$\frac{\rho V^2}{2} = \frac{\delta_p \rho g \ell^2}{8}\left(-\frac{1}{c_p}\frac{ds}{dz}\right)$$

2) Definition of dimensionless *superadiabaticity*, a local measure of instability:

$$\Delta\nabla = -\frac{H}{c_p}\frac{ds}{dz}$$

Density changes caused by pressure perturbations are neglected. Using the previous estimate of F_{conv} that also relates V and ds/dz,

$$\boxed{M = U_1(\Delta\nabla)^{1/2} = U_2\frac{(F_{\text{conv}}/\rho)^{1/3}}{c_s}}$$

$$U_1 \equiv \left[\frac{a^2\delta_p\alpha_T}{4\gamma}\right]^{1/2} \sim O(1), \qquad U_2 \equiv \left[\frac{a\delta_p r_*}{2c_p}\right]^{1/3} = \left[\frac{a\alpha_T(\gamma-1)}{2\delta_p\gamma}\right]^{1/3} \sim O(1)$$

The Mach number $M = V/c_s$. For an ideal gas, $\gamma = 5/3$, $\alpha_T = 1$, and $\delta_p = 1$.

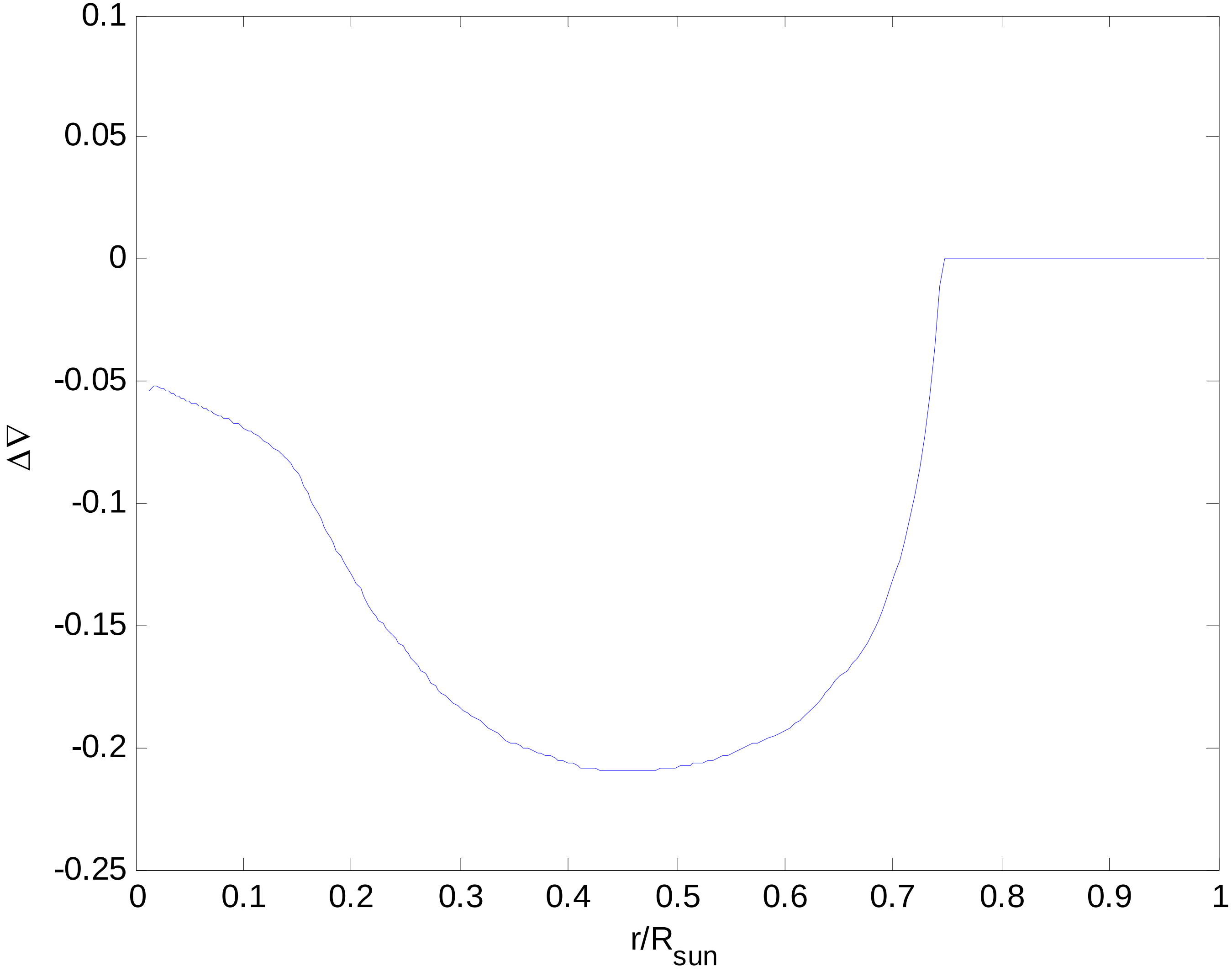
Scales of Solar Convection

Type of observation	Granules	Meso-granules?	Super-granules	Giant cells?
Horizontal velocity	1400 m/sec	?	300 m/sec	25 m/sec
Vertical velocity	900 m/sec	60 m/sec	35 m/sec	?
Horizontal length scale	1.4 Mm	7.0 Mm	30 Mm	340 Mm
Cell lifetime	0.07 hours	2.0 hours	25 hours	1800 hours
Turnover time	0.30 hours	?	30 hours	3800 hours
theory				
Squared Mach #	0.11	?	1.4×10^{-5}	1.6×10^{-7}
Rossby #	167	?	1.67	0.012

Jupiter's Great Red Spot: 0.015

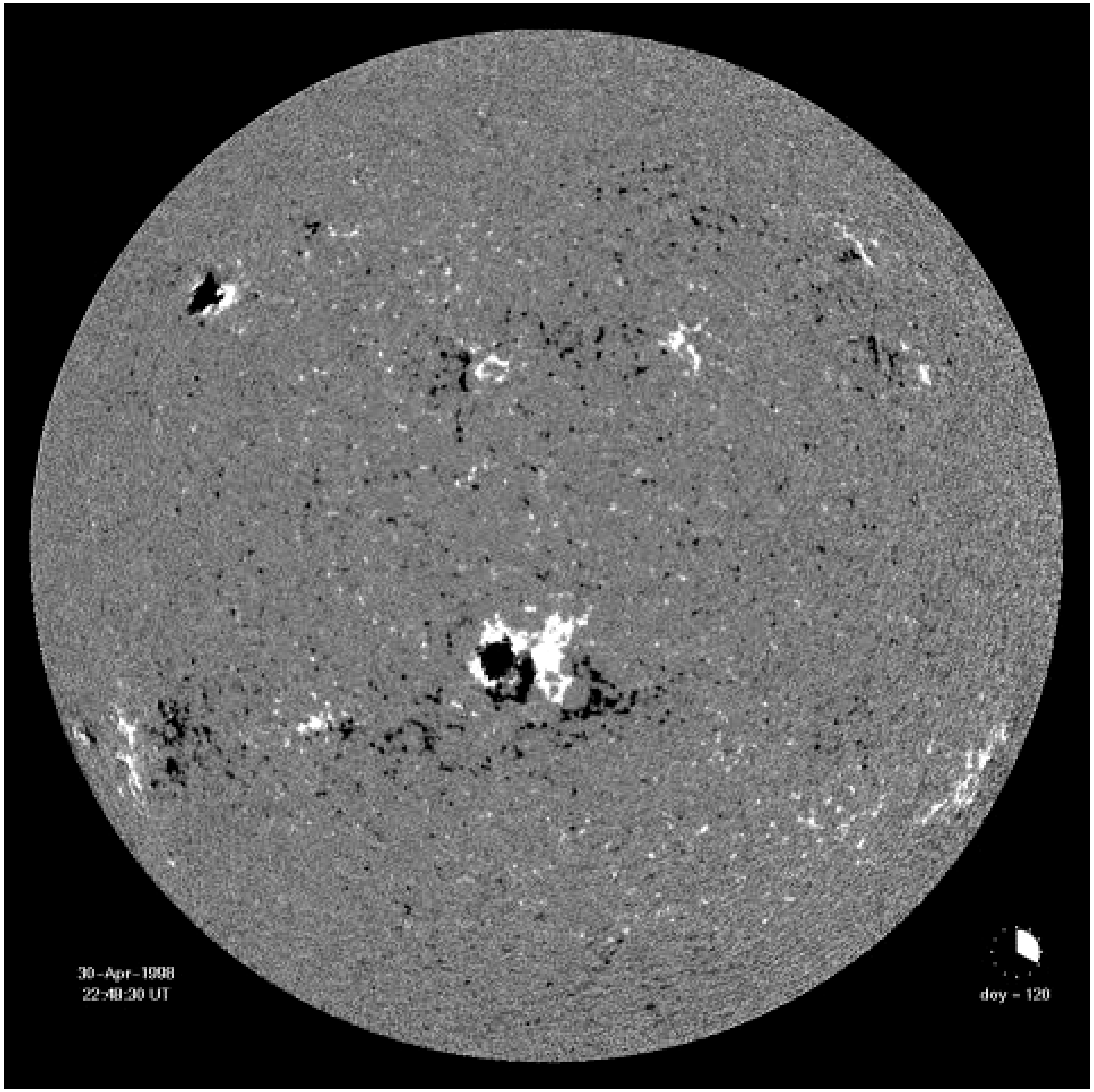
Result: Superadiabaticity in the Sun

Note, it's essentially zero in the convection zone



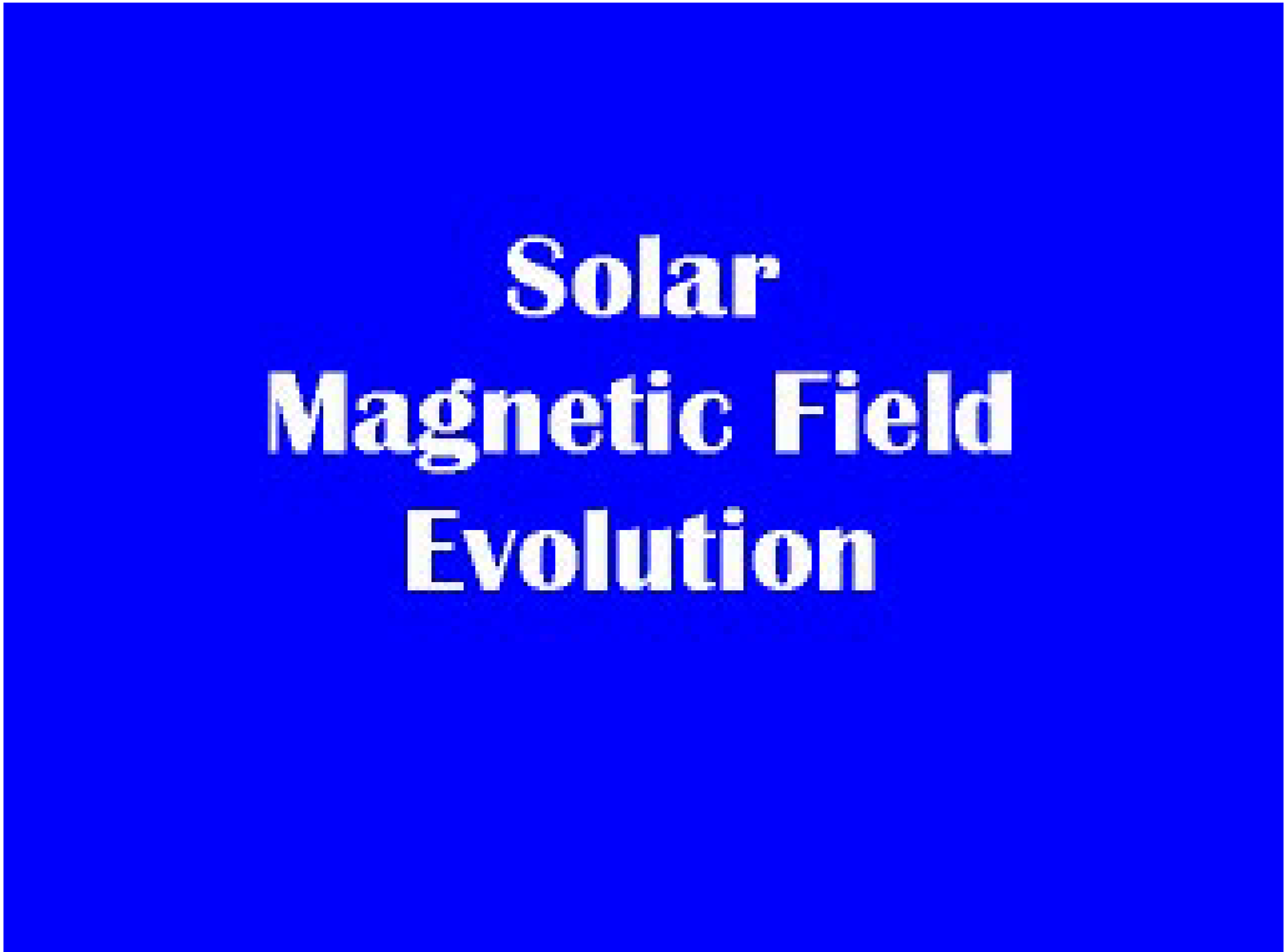
Animation of Magnetograms

Magnetic fields trace out patterns of convection and rotation



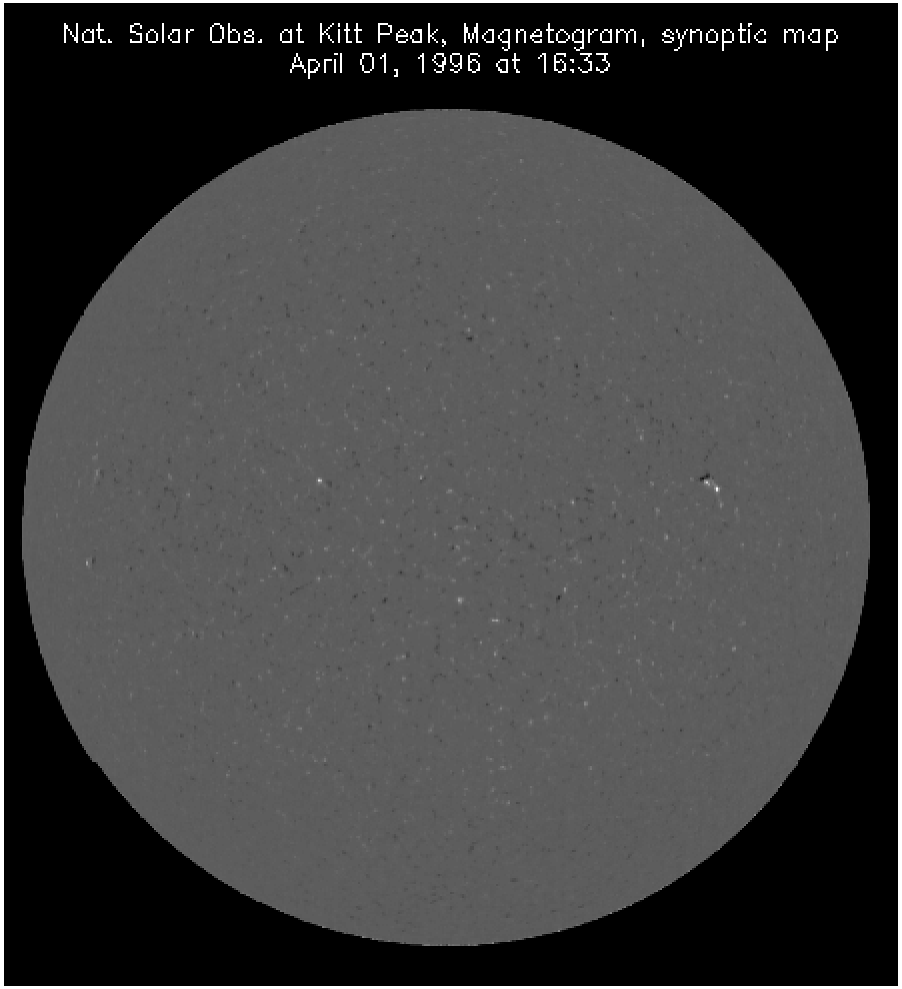
**Play the QuickTime version to speed it up*

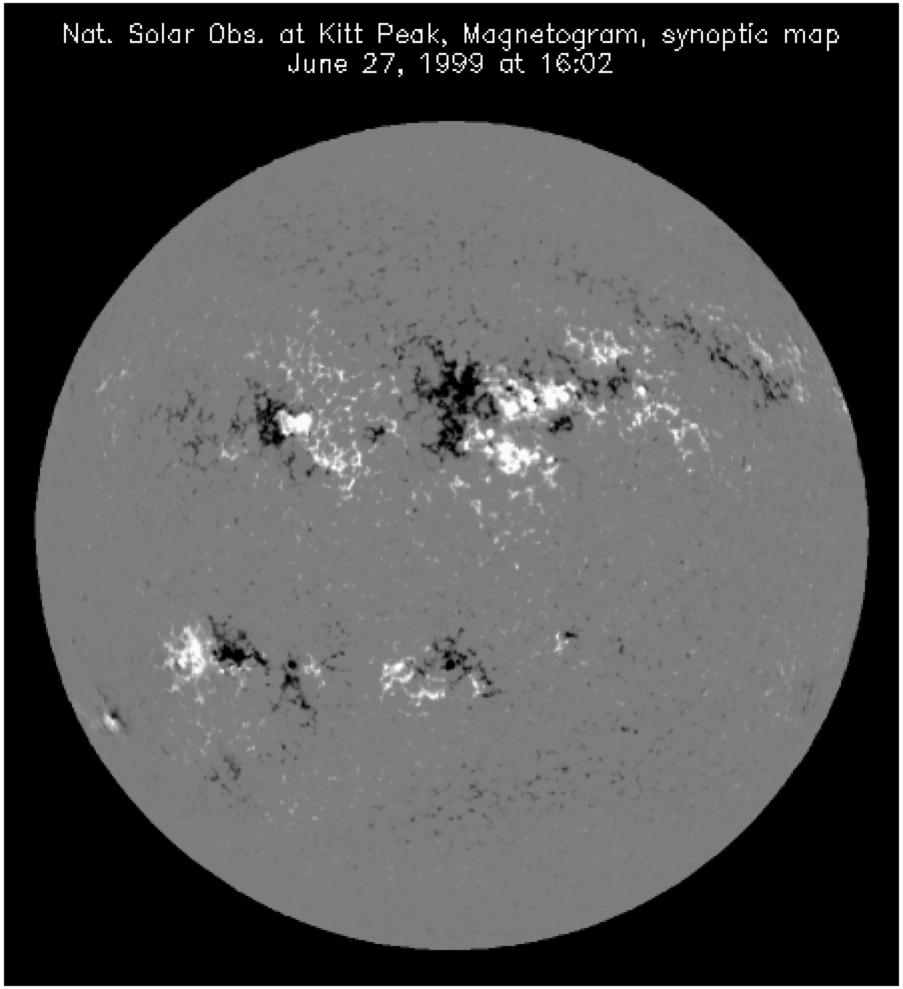
Magnetic Field Evolution over Two Solar Cycles



**QuickTime version has better resolution*

Solar Min vs. Solar Max…





What is responsible for this *periodic re*-generation of the Sun's magnetic fields?
It seems the mechanism, whatever it is, must be global in scope

Dynamo Theory and Magnetohydrodynamics – 1

Starting from the single-fluid equations for ions and electrons

$$m_j N_j \frac{\partial \underline{v}_j}{\partial t} = q_j N_j \left(\underline{E} + \frac{\underline{v}_j \times \underline{B}}{c} \right) - \nabla p_j + m_j N_j \underline{g}$$

Let $\rho = \sum_{j=1}^{2} m_j N_j$, $\quad \underline{v} = \frac{1}{\rho} \sum_{j=1}^{2} m_j N_j \underline{v}_j$

$\underline{j} = \sum_{j=1}^{2} q_j N_j \underline{v}_j$. Note that if $N_i \approx N_e$, $q_i = -q_e$;

$\rho = N(m_i + m_e)$, $\quad \underline{v} = \frac{1}{\rho} N(m_i \underline{v}_i + m_e \underline{v}_e) = \frac{m_i \underline{v}_i + m_e \underline{v}_e}{m_i + m_e}$

$\underline{j} = eN(\underline{v}_i - \underline{v}_e)$ Add equations for ions, electrons

$$N \frac{\partial}{\partial t}(m_i \underline{v}_i + m_e \underline{v}_e) = \frac{eN}{c}(\underline{v}_i - \underline{v}_e) \times \underline{B} - \nabla p + \rho \underline{g}$$

$$\rho \frac{\partial \underline{v}}{\partial t} = \frac{1}{c} \underline{j} \times \underline{B} - \nabla p + \rho \underline{g}$$

neglect viscosity + nonlinear term; also ion-e^- collision

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Magnetohydrodynamics – 2

The magnetic induction equation

Know: Ohm's Law $\underline{j} = \sigma \underline{E}'$ in rest frame of the moving fluid element

$$\boxed{\underline{j} = \sigma\left(\underline{E} + \frac{\underline{v}}{c}\times\underline{B}\right)} \longleftarrow$$ in fixed frame of ref. (do a Lorentz transformation)

OHM'S LAW

Add Ampère's Law $\nabla\times\underline{B} = \frac{4\pi}{c}\underline{j} + \cancel{\frac{1}{c}\frac{\partial \underline{E}}{\partial t}}$ → neglect - low freq.

(neglecting magnetizability, polarizability of medium)

Faraday's Law $\nabla\times\underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t}$

$$\frac{c}{4\pi\sigma}\nabla\times\underline{B} = \underline{E} + \frac{\underline{v}}{c}\times\underline{B}, \qquad \nabla\times\left(\frac{c^2}{4\pi\sigma}\nabla\times\underline{B}\right) = -\frac{\partial \underline{B}}{\partial t} + \nabla\times(\underline{v}\times\underline{B})$$

Let $\eta \equiv \frac{c^2}{4\pi\sigma}$

$$\boxed{\frac{\partial \underline{B}}{\partial t} = \nabla\times(\underline{v}\times\underline{B}) - \nabla\times(\eta\,\nabla\times\underline{B})}$$

MHD Ohm's Law or magnetic induction eqn.

Magnetohydrodynamics – 3

Alternate forms for the magnetic terms

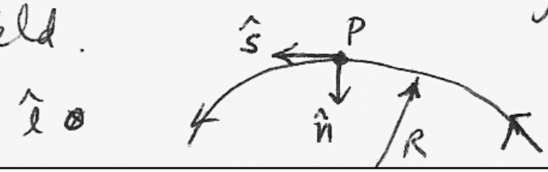
If $\eta =$ const., $\quad \underbrace{\frac{\partial \underline{B}}{\partial t}}_{\text{material}} = \underbrace{\nabla \times (\underline{v} \times \underline{B})}_{\text{advection}} + \underbrace{\eta \nabla^2 \underline{B}}_{\text{diffusion}}$

If $\nabla \cdot \underline{v} = 0$, $\quad \underbrace{\frac{\partial \underline{B}}{\partial t} + \underline{v} \cdot \nabla \underline{B}}_{\text{advective deriv.}} = \underbrace{\underline{B} \cdot \nabla \underline{v}}_{\text{stretching}} + \underbrace{\eta \nabla^2 \underline{B}}_{\text{diffusion}}$

Next look at Lorentz force.

$$\frac{1}{4\pi} (\nabla \times \underline{B}) \times \underline{B} = \underbrace{-\nabla \left(\frac{B^2}{8\pi} \right)}_{\text{pressure}} + \underbrace{\frac{1}{4\pi} (\underline{B} \cdot \nabla) \underline{B}}_{\text{tension}} = \underbrace{-\nabla_\perp \frac{B^2}{8\pi}}_{\text{perp. pressure}} + \underbrace{\frac{B^2}{4\pi} \frac{\partial \hat{b}}{\partial s}}_{\text{curvature}}$$

Look a little more at Lorentz force in a curved magnetic field.



constant radius of curvature R

Magnetohydrodynamics – 4

Interpretation of the Lorentz force

So $\underset{\sim}{B} = B\hat{s}$ and $(\underline{B}\cdot\nabla)\underline{B} = B\frac{\partial}{\partial s}(B\hat{s}) = B\frac{\partial B}{\partial s}\hat{s} + B^2\frac{\partial \hat{s}}{\partial s}$

$\frac{\partial\hat{s}}{\partial s}$ = rate of change in pointing dir. of field line as you move along it

Think of plane polar coords; $\Delta\hat{s} = \hat{n}\,\Delta\theta$, $\Delta s = R\,\Delta\theta$

Infinitesimal limit $\frac{\Delta\hat{s}}{\Delta s} \to \frac{\partial\hat{s}}{\partial s} = \frac{\hat{n}}{R}$

$\therefore\ (\underline{B}\cdot\nabla)\underline{B} = B\frac{\partial B}{\partial s}\hat{s} + \frac{B^2}{R}\hat{n} = \hat{s}\frac{\partial}{\partial s}\left(\frac{B^2}{2}\right) + \hat{n}\frac{B^2}{R}$

Plug into $-\nabla\frac{B^2}{8\pi} + \frac{1}{4\pi}(\underline{B}\cdot\nabla)\underline{B}$, Lorentz force:

$$-\hat{n}\frac{\partial}{\partial n}\left(\frac{B^2}{8\pi}\right) - \hat{l}\frac{\partial}{\partial l}\left(\frac{B^2}{8\pi}\right) \cancel{- \hat{s}\frac{\partial}{\partial s}\left(\frac{B^2}{8\pi}\right) + \hat{s}\frac{\partial}{\partial s}\left(\frac{B^2}{8\pi}\right)} + \hat{n}\frac{B^2}{4\pi R}$$

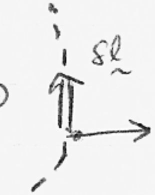
Notice the s-components cancel: force is $\perp$ $\underline{B}$ ← as it derives

Frozen-In Flux

MHD in the limit of zero resistivity

Now: how to interpret magnetic induction eqn.?
Consider magnetic flux Φ across arbitrary surface Σ moving with velocity $\underline{V}(\underline{r})$ (not uniformly)

$$\frac{d\Phi}{dt} = \underbrace{\iint_\Sigma \frac{\partial \underline{B}}{\partial t} \cdot d\underline{a}}_{\text{change due to overall change in } \underline{B}} + \underbrace{\oint \underline{B} \cdot (\underline{V} \times d\underline{\ell})}_{\text{change due to border of } \Sigma \text{ either growing or shrinking}}$$



But $\oint \underline{B} \cdot (\underline{V} \times d\underline{\ell}) = -\oint (\underline{V} \times \underline{B}) \cdot d\underline{l} = -\iint_\Sigma \nabla \times (\underline{V} \times \underline{B}) \cdot d\underline{a}$

Stokes' theorem

$$\therefore \frac{d\Phi}{dt} = \iint_\Sigma \left[\frac{\partial \underline{B}}{\partial t} - \nabla \times (\underline{V} \times \underline{B}) \right] \cdot d\underline{a}$$

From MHD induction: $\sigma \to \infty \Rightarrow \eta \to 0 \Rightarrow \frac{d\Phi}{dt} = 0$

$\therefore$ Infinite conductivity $\Rightarrow$ "frozen-in" flux travels along with fluid. Finite cond. means slippage can occur.
Note $-\nabla \times (\eta \nabla \times \underline{B}) = \eta \nabla^2 \underline{B}$: DIFFUSION for η = const.

This is how differential rotational is able to "wind up" a seed field

Dimensionless Numbers of MHD

Scale $\mathbf{v}$ by V, ∇ by $\frac{1}{L}$, $\mathbf{B}$ by B_0 in induction eqn.

$$\mathbf{v}\cdot\nabla\mathbf{B} \sim \mathbf{B}\cdot\nabla\mathbf{v} \sim \frac{B_0 V}{L} \; ; \quad \eta\nabla^2\mathbf{B} \sim \eta\frac{B_0}{L^2}$$

Ratio = $\boxed{\frac{VL}{\eta} \equiv \text{magnetic Reynolds \#, } R_m}$ $\boxed{\text{BIG FOR } L = \text{LARGE (astro.)}}$

dimensionless Ohm's Law, $\frac{\partial\mathbf{B}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{v} + \frac{1}{R_m}\nabla^2\mathbf{B}$

Next, $\rho\mathbf{v}\cdot\nabla\mathbf{v} \sim \rho_0\frac{V^2}{L}$ $\quad \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} \sim \frac{1}{4\pi}\frac{B_0^2}{L}$ in mmtm. eqn.

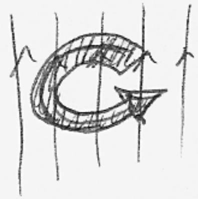
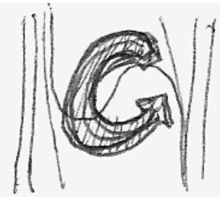
Ratio = $\frac{V^2 \cdot 4\pi\rho_0}{B_0^2} = \boxed{\left(\frac{V}{V_A}\right)^2 = [\text{Alfvén Mach \#}]^2}$

Alternatively, $-\nabla p \sim \frac{p_0}{L}$; $\boxed{\frac{p_0}{B_0^2/8\pi} \equiv \beta}$ $\boxed{\frac{V}{\sqrt{p_0/\rho_0}} \sim M}$

PLASMA BETA; MACH NO.

Plasma β is also the square of the ratio of sound speed to Alfvén speed.
Large Rm means **B** will get pulled and squeezed until it can fight back…

Flux expulsion: like a fluid boundary layer in 2D

 becomes 

If this is strong ⇒ get nonlinear response.

☆ [SEE ARGUMENT NEXT PG.] -23-

Equipartition at photosphere: $p \sim \frac{B^2}{8\pi}$ (Zirin, p. 126)

Region	Height	Density	Temp	NkT	⇒ B (gauss)	⇒ V_A ($\frac{km}{s}$) lower bound
Photosphere	0	10^{17}	6000	8.3×10^4	1440	10
Chromosphere	1500	10^{12}	10^4	2.8	8.4	18.5
Corona	3000	10^8	10^6	0.028	0.84	185

Average observed field in photosphere is about 1 Gauss. But sunspots ≳ 1 kG. Current belief: all fields are ~ kG, apparently weak ones aren't spatially resolved.

Magnetic Buoyancy Instability

<u>Magnetic</u> <u>Buoyancy</u> in Convection Zone. (Eugene Parker)

If $p \gtrsim \frac{B^2}{8\pi}$, then density reduction is significant inside high-B region (thermo. p down $\Rightarrow$ fluid density down, relative to field-free surroundings).

In gravity field: high-B regions are buoyant!

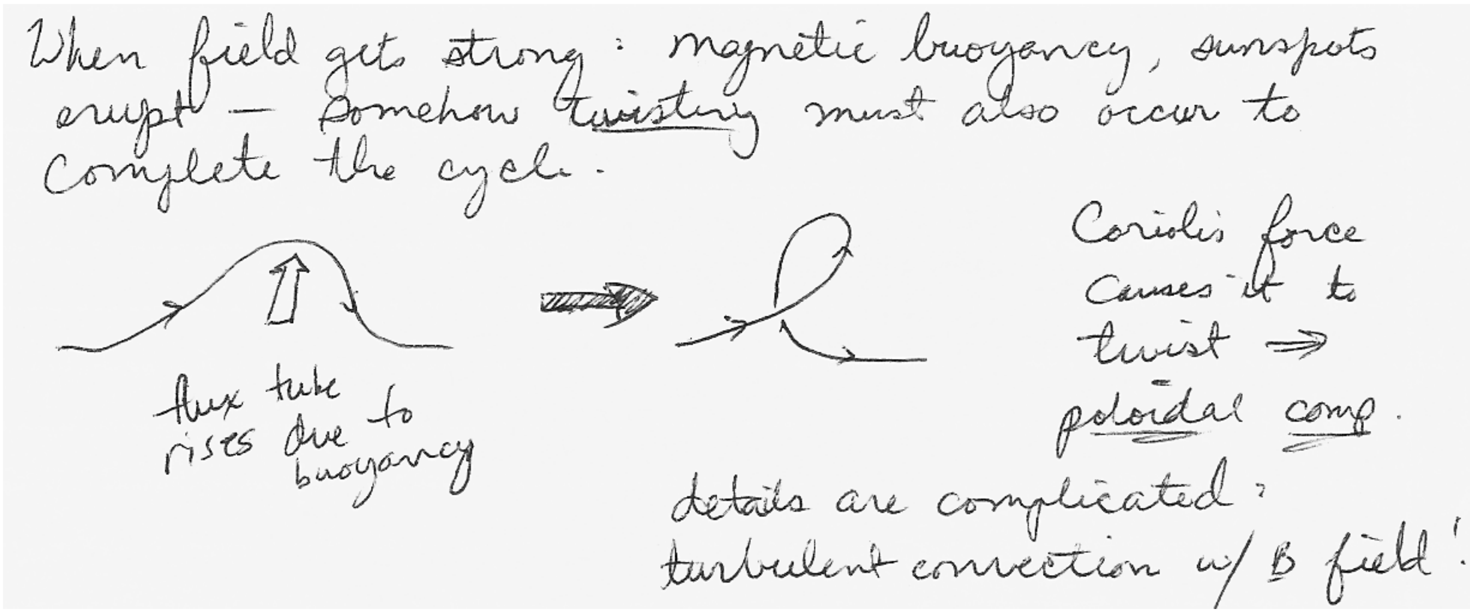
etc.

This is probably why flux emerges in the first place.

<u>Magnetic</u> <u>Tension</u> competes with buoyancy; a very difficult problem!

In the convection zone, $B^2/8\pi$ only has to be comparable to $\rho T\Delta s \sim \rho T c_p M^2$

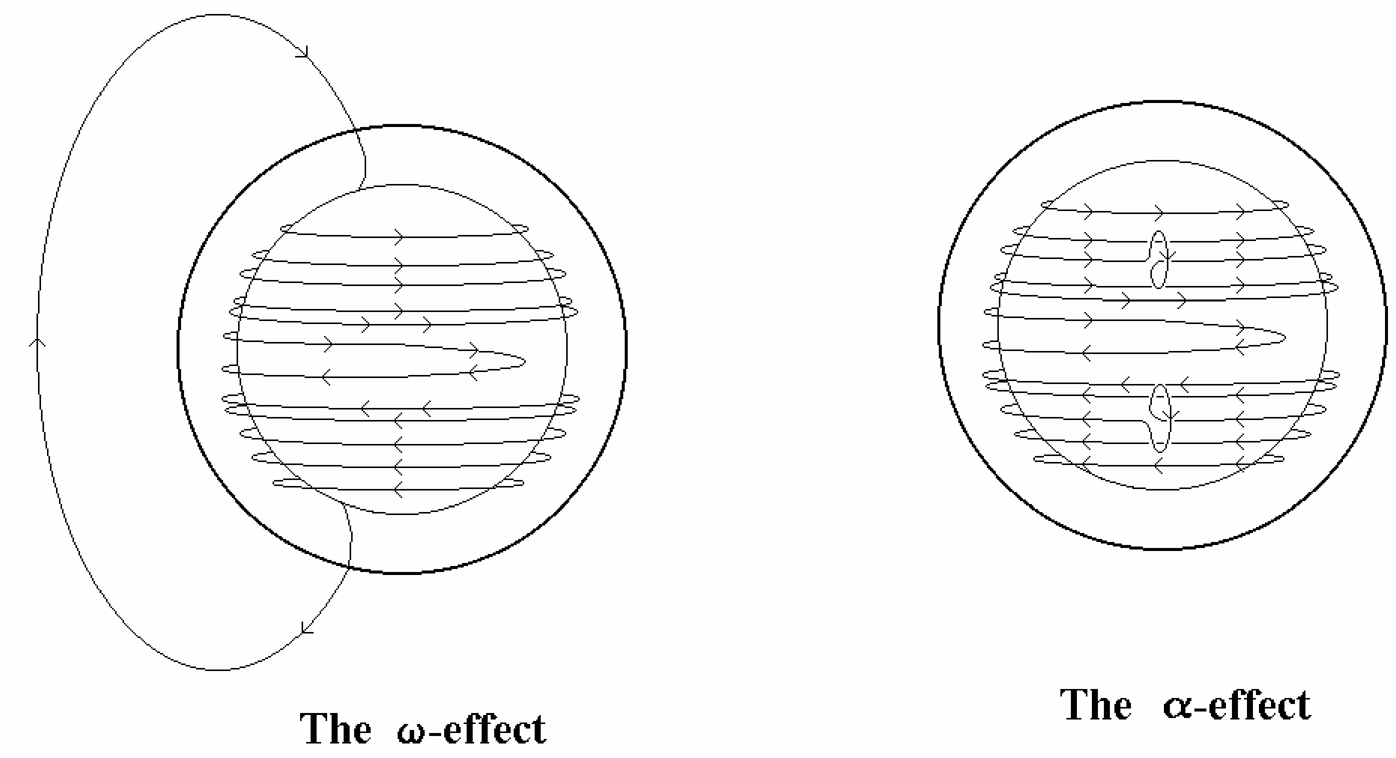
Magnetic Buoyancy with a Twist



Points that rise radially (diverge) tend to acquire an anticyclonic spin. Surprisingly, this rather simplistic picture of a rising, twisting flux tube not only provides an essential "α**-effect**" for dynamo theory, it can be shown to satisfy both **Joy's Law** and **Hale's Polarity Law**…

The Dynamo Must Have an α- as Well as an ω-effect

For a full solar cycle, fields must be converted back to poloidal



poloidal $\rightarrow$ toroidal field

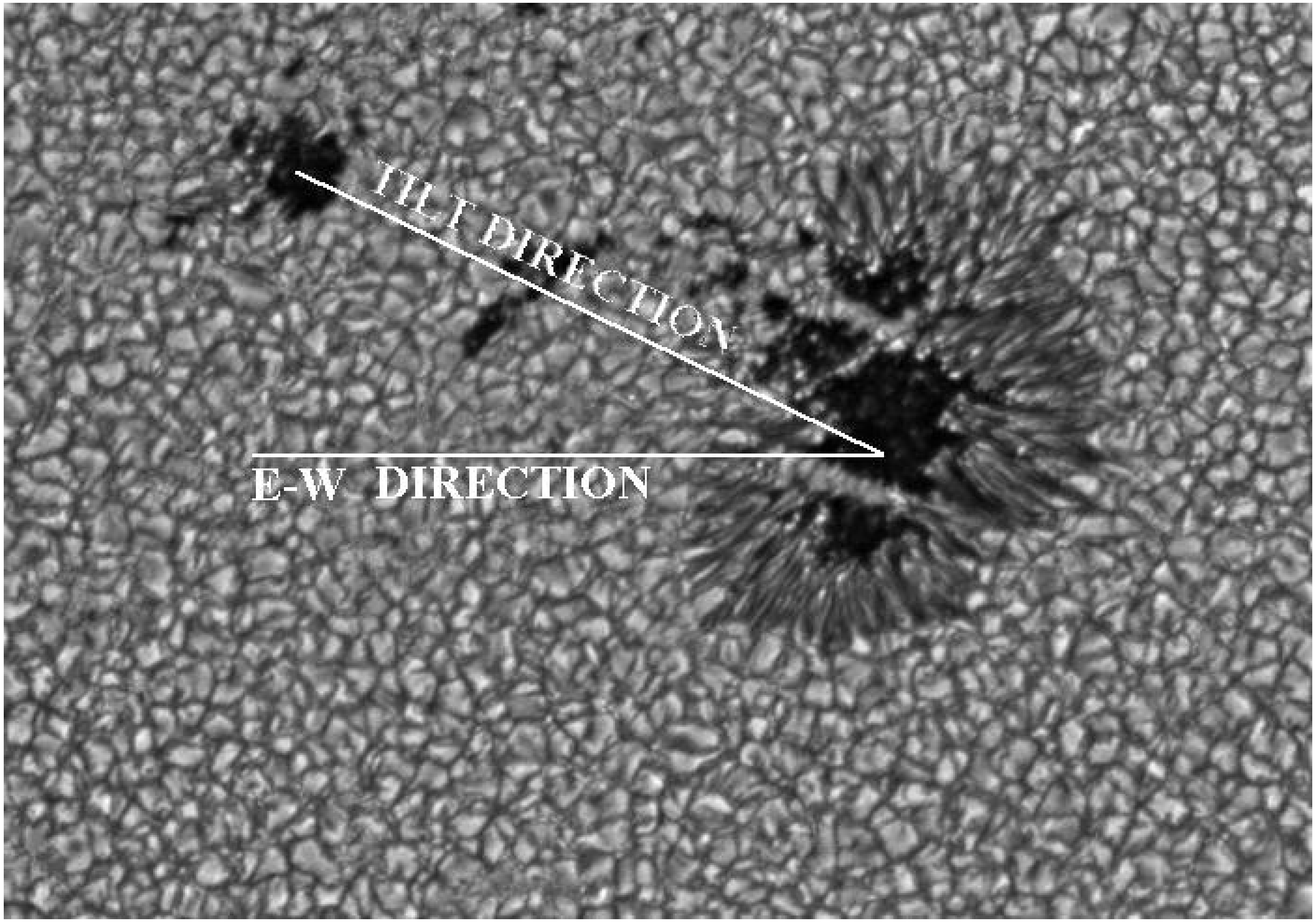
(bar magnet $\rightarrow$ 2 donuts)

toroidal $\rightarrow$ poloidal field

(2 donuts$\rightarrow$ bar magnet)

Joy's Law

The leading sunspot of a pair is always tilted toward the equator

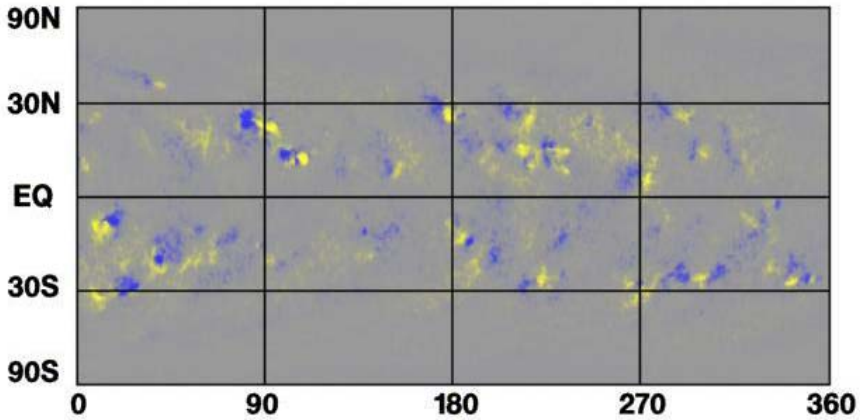


A rising flux tube will tend to tilt like this due to Coriolis forces

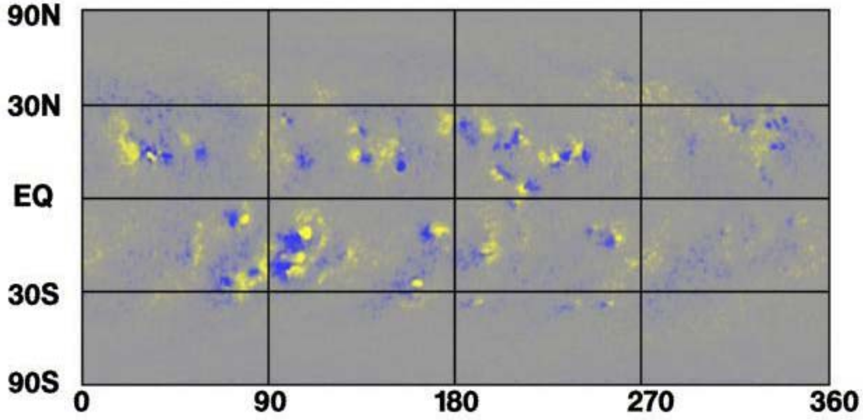
Hale’s Polarity Law

The polarity of the leading spots in one hemisphere is opposite that of the leading spots in the other hemisphere and the polarities reverse from one cycle to the next.

Cycle 21

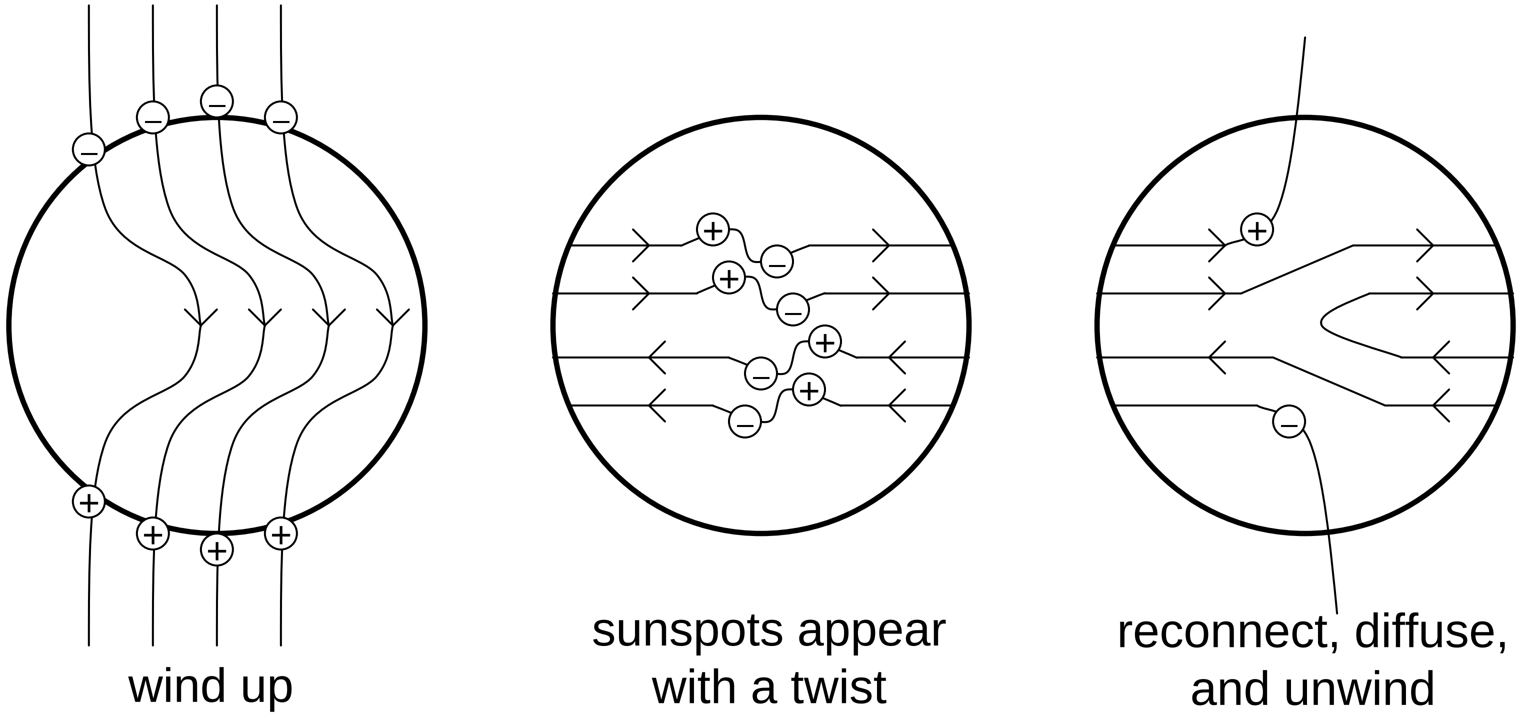


Cycle 22



Putting It All Together…

A cartoon of how the solar cycle might possibly work?



- Equatorward migration of sunspots (called “Spörer’s Law,” though first noted by Carrington around 1861) is explained by the return flow of the meridional circulation, deep in the convection zone
- Poleward migration of flux away from sunspots (via diffusion plus the surface meridional flow) creates opposite polarity for the next cycle

Rival α-effect Based on Mean Field Dynamo Theory

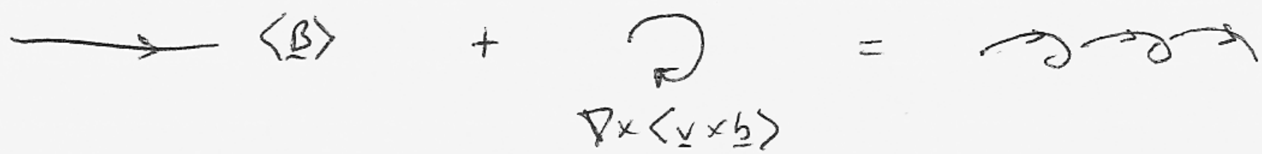
α-effect: Mean field dynamo theory

$$\underline{B} = \langle \underline{B} \rangle + \underline{b}$$

mean part $$\frac{\partial \langle \underline{B} \rangle}{\partial t} = \nabla \times \left(\langle \underline{V} \rangle \times \langle \underline{B} \rangle \right) + \eta \nabla^2 \langle \underline{B} \rangle + \nabla \times \left(\langle \underline{v} \times \underline{b} \rangle \right)$$

correlation of fluctuations $\Rightarrow$ EMF.

Assumption: $\langle \underline{v} \times \underline{b} \rangle = \alpha \langle \underline{B} \rangle$ $\Rightarrow$ current is driven along the mean field ("magnetic helicity")



Problem: simulations designed to test this theory show that the magnetic energy tends to build up at small, turbulent scales instead of large ones